Analysis of Seismic Response of PC Continuous Beam Bridges

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Abstract
To improve the crack resistance of concrete, and increase the stiffness and durability of the structures the prestressed structure are used widely in all kinds of engineering structures in recent years. The prestressed structure can make full use of the high strength of the material properties and has significant social and economic benefits. This paper proceeds the analysis of a PC continuous T-shaped bridge which is carried out by ANSYS. A three-dimensional finite element model is established. The characteristics of seismic response is studied. The results show that the displacement in vertical direction of the bridge is much larger than those in the other two directions, and the closer to the middle position the greater the vertical displacement. The conclusions can provide reference for the seismic design and construction of prestressed concrete bridges.

Key words: Prestress, Dynamic Analysis, Seismic Response, Seismic Design.

1. INTRODUCTION

Bridge is an important component of lifeline system, and it is also the hub of transportation. In the event of an earthquake, if the bridges were destroyed, especially for the viaducts and highway trunks in city, the disaster would cause incalculable damage to the city traffic and post disaster rescue (Yao LingSen, 1997; Fan Lichu, 1997). Due to the span of bridges and the development of highway bridges increased rapidly, seismic performance of the bridge must be guaranteed. At present, the code for seismic design of the bridge is not complete and the scope of application is limited, the research work in this area needs to be further studied (Fan and Wang, 2001). Therefore, it is of great practical significance to strengthen and enrich the research on the seismic theory of bridges. To study the vibration regular pattern of bridge structure under earthquake load, provide theoretical basis for seismic design of bridges, and to maintain the safe operation of the bridge, it has an important preventive effect on the safety of the bridge structure.

Since 1899, the Japanese scholar Masanori Mori first proposed for the seismic design of the static method, the seismic response analysis of the bridge structure has gone through a static method, dynamic response spectrum method and dynamic time history analysis of three stages. Due to the static method neglects the important factor of the dynamic characteristics of the structure, the seismic acceleration is regarded as the single factor of structural earthquake damage, so it has very big limitation. It is only applicable to the structure with large stiffness (Hu, 1988). And the response spectrum method can only get the maximum response and can not reflect the structure in the process of ground motion experience, and therefore can not give the order of destruction of various components, it is difficult to determine the structure of the damage mechanism (Hasan, Xu and Grierson, 2002).

According to the dynamic time history analysis method, the continuous structure is discretized into a multi node, multi degree of freedom system, the finite element dynamic equation is established, and the acceleration of ground motion is directly input, and the response of the structure is calculated. It can deal with the nonlinear dynamic response of complex bridges under earthquake action, and get the dynamic response (velocity, acceleration, stress, etc.) of the structure during the whole time of the earthquake. So the dynamic time history analysis is the only feasible analytical method for the nonlinear seismic response of the bridge. It is a hot spot and development trend of bridge seismic research(Fang and Hu, et al., 2001).

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2. BASIC THEORY OF FINITE ELEMENT METHOD

Finite element method appeared in the middle of 1950s. It is a new numerical method. It solves many complex boundary problems. Its essence is to discrete and interpolation as the guiding ideology, a continuous structural system is discretized into a finite element, and these elements are connected in a certain way. Then a complex multi degree of freedom system is transformed into a discrete finite degree of freedom system (Wang ZhuCheng and Shao Ming, 1997).

For the dynamic problem, the load of the structure is changed with time (Wang, 2009; Zhang and Algor, 2005). According to the finite element method, the structure of the load distribution to the node, get the function of nodal load matrix \( \{ F \} \) is also a time function \( \{ F(\tau) \} \). Structure in motion, the points in addition to displacement, \( \{ f \} \) there is speed \( \{ \dot{f} \} \) and acceleration \( \{ \ddot{f} \} \). In accordance with the Dahlber principle, the acceleration of the mass should be added inertial load. If the density of the material is \( \rho \), the inertia force of the structure unit volume is \(-\rho \{ \ddot{f} \}\). This structure is subject to another volume distribution load, the size and the point is proportional to the acceleration, and the direction of the acceleration in the opposite direction. In addition, the particle in the movement process will be resistance. If the unit volume material resistance coefficient is \( \mu \), and is set and the speed is proportional, the resistance of unit volume is \(-\mu \{ \dot{f} \}\). The displacement function is \( [N] \).

According to the static finite element method, the displacements of the elements are expressed by the displacement of element nodes \( \{ \delta \} \).

\[
\{ f \} = [N]\{ \delta \}^r
\]  
(1)

The speed and acceleration in the unit are respectively:

\[
\{ \dot{f} \} = [N]\{ \dot{\delta} \}^r
\]  
(2)

\[
\{ \ddot{f} \} = [N]\{ \dddot{\delta} \}^r
\]  
(3)

where \( \{ \dot{\delta} \}^r \) and \( \{ \dddot{\delta} \}^r \) are the velocity and acceleration arrays of the unit nodes. The unit inertial force \(-\rho \{ \ddot{f} \}\) and resistance \(-\mu \{ \dot{f} \}\) as the volume distribution load assigned to each node, you can get the unit node load, denoted by \( \{ F \}_\rho \) and \( \{ F \}_\mu \).

\[
\{ F \}_\rho = -\int [N]^T \rho [N] d\{ \delta \}^r dV = -\int [N]^T \rho [N] \{ \ddot{\delta} \}^r dV = -[m]\{ \dot{\delta} \}^r
\]  
(4)

\[
\{ F \}_\mu = -\int [N]^T \mu [N] d\{ \delta \}^r dV = -\int [N]^T \mu [N] \{ \dot{\delta} \}^r dV = -[c]\{ \dot{\delta} \}^r
\]  
(5)

where \([m]\) is the element mass matrix defined as

\[
[m] = \int [N]^T \rho [N] dV
\]  
(6)

and the element damping matrix is

\[
[c] = \int [N]^T \mu [N] dV
\]  
(7)

The balance equation is:

\[
[m][\dot{\delta}]^r + [c][\dot{\delta}]^r + [k][\delta]^r = \{ F \}^r
\]  
(8)

where \([k]\) is the element stiffness matrix.

According to the finite element method, the dynamic equilibrium equation of the structure is obtained as

\[
[M][\ddot{\delta}] + [C][\dot{\delta}] + [K][\delta] = \{ F \}
\]  
(9)

in the above equation, \([M]\)—System quality matrix, \([C]\)—Damping matrix of the system.
3. MODAL ANALYSIS THEORY

The modal analysis is also known as the natural vibration model analysis. It is mainly used to determine the vibration characteristics of the structure, such as the natural frequencies of the structures and the modes of vibration, which are the important parameters in the design of structural bearing dynamic loads (Tang Wei, 2011).

The benefits of modal analysis include:
1. Avoid resonance or vibration at a specific frequency in the structural design
2. Make engineers recognize the structure for different types of dynamic load is how to respond;
3. Helps to estimate the solution control parameters in other dynamic analyzes.

According to the basic theory of finite element, the elastic dynamic equations of the whole system are:

\[ [M][\ddot{\delta}]+[C][\dot{\delta}]+[K][\delta]=\{F\} \]

The meaning of each symbol in the formula see formula(9), the damping and external force for the free vibration problem of the non-damped free vibration are 0. Thus, the dynamic equation is changed to:

\[ [M][\ddot{\delta}]+[K][\delta]=0 \]  

(10)

This is a constant coefficient homogeneous linear differential equations, the solution of the form:

\[ \{\delta(t)\} = \{\phi\} \sin wt \]  

(11)

The formula (10) derived (11) generalized characteristic equation:

\[ ([K]-\omega^2[M])\{\phi\} = 0 \]  

(12)

The formula is homogeneous linear algebraic equations, and the condition of nonzero solutions is that the determinant is equal to zero.

\[ \begin{vmatrix} [K] & -\omega^2[M] \end{vmatrix} = 0 \]  

(13)

If the order of \([K]\) and \([M]\) is \(n\), then the formula(13) is \(n\) equation \(\omega^2\), called structural free vibration characteristic equation, \(\omega^2\) called eigenvalues. The equation can be solved for \(n\) eigenvalues, into the (12), can be solved \(n\) values of \(\{\phi\}\), \(\{\phi\}\) known as eigenvectors. The \(i\)-th \(\omega\) and \(\{\phi\}\) are the \(i\)-th natural frequency of the structure and the \(i\)-th mode of vibration, respectively.

4. TIME-HISTORY ANALYSIS OF FINITE ELEMENT DYNAMIC ANALYSIS

This method is selected from a more suitable geological conditions as the input parameters of the vibration wave, with multi-degree-of-freedom structure, used the integral method to solve the equation (Xie Xu, 2006).

Generally take the equidistant time interval on the actual calculation, from the initial moment \(t_0 = 0\) to a specified moment \(t_n = T\). Solution of dynamic equilibrium equation by step integral. Make the solve domain of time \([0,T]\), equal divided into \(n\) time interval \(\Delta t = \frac{T}{n}\), after assume the initial moment of displacement, velocity and acceleration, to find the displacement, velocity and acceleration of the \(t\) moment, and then gradually solve the solution of \(t, t+\Delta t\) time, the process of the calculation is to find the solution of \(t + \Delta t\) time, use this solves the process to build up a general algorithm for solving all discrete time solutions. At present, the most commonly used time-history analysis methods in structural seismic response analysis are central difference method, linear acceleration method, Wilson method and Newmark method.

The integrated calculation steps are as follows:

1. Initial calculation
   1. Form the stiffness matrix \([K]\), the mass matrix \([M]\) and the damping matrix \([C]\).
   2. Get the initial value \(\{\delta_0\}, \{\dot{\delta}_0\}, \{\ddot{\delta}_0\}\)
   3. Select the step size \(\Delta t\), parameter sum \(\gamma\) and \(\beta\) calculate the following parameters
      \[
      \gamma \geq 0.50 \quad \beta \geq 0.25(0.5 + \gamma)^2
      \]
\[ a_0 = \frac{1}{\beta(\Delta t)^2}, \quad a_1 = \frac{\gamma}{\beta \Delta t}, \quad a_2 = \frac{1}{2\beta} - 1, \quad a_3 = \frac{\gamma}{\beta} - 1, \quad a_4 = \frac{\Delta t}{2}(\frac{\gamma}{\beta} - 2), \quad a_5 = \Delta T(1 - \gamma), \]

\[ a_7 = \gamma \Delta t \]

④ A stiffness matrix is formed \[ [\hat{K}] = [K] + a_0 [M] + a_1 [C] \]

⑤ Inverse matrix \[ [\hat{K}]^{-1} \]

⑵ For each time step calculation

① The load column vector at time \( t + \Delta t \) is calculated

\[ \{\hat{F}_{t+\Delta t}\} = [F_{t+\Delta t}] + [M]\{\delta_t\} + a_2 \{\dot{\delta}_t\} + a_3 \{\ddot{\delta}_t\} + [C]\{\delta_t\} + a_4 \{\dot{\delta}_t\} + a_5 \{\ddot{\delta}_t\} \]

② Solve the displacement at time \( t + \Delta t \)

\[ \{\delta_{t+\Delta t}\} = [\hat{K}]^{-1}\{\hat{F}_{t+\Delta t}\} \]

③ Solve the acceleration, speed at time \( t + \Delta t \)

\[ \{\ddot{\delta}_{t+\Delta t}\} = a_4 \{\ddot{\delta}_{t+\Delta t}\} - a_2 \{\dot{\delta}_t\} - a_3 \{\dot{\delta}_t\} \]

\[ \{\ddot{\delta}_{t+\Delta t}\} = a_5 \{\ddot{\delta}_t\} + a_7 \{\ddot{\delta}_t\} + a_6 \{\dot{\delta}_t\} \]

5. BRIDGE ANALYSIS MODEL

5.1. Engineering Data

In this paper, the research object is the PC continuous T-shaped beam bridge of the Lian Huo highway. According to the design requirements of the highway, the upper part of the bridge is three span continuous girder bridge, and the span is 29.92m, 30m, 29.92m. The width of bridge deck is \( 2 \times (10 + 0.5 + 0.75) \). Design load is highway - I level, the peak of the earthquake acceleration in the area of bridge is \( 0.15g \).

Calculations are assumed as follows:

(1) The material of the bridge model is uniform;

(2) Line elastic constitutive model is used in bridge model.

5.2 Modal Analysis

For bridges, generally take 5 to 10 order on it. And the frequency of the first 10 steps is calculated as follow:

<table>
<thead>
<tr>
<th>Order</th>
<th>Frequency (Hz)</th>
<th>The vibration mode shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.236</td>
<td>Y-axis direction of the vibration-based</td>
</tr>
<tr>
<td>2</td>
<td>5.645</td>
<td>Y-axis direction of the main twist</td>
</tr>
<tr>
<td>3</td>
<td>10.655</td>
<td>X-axis direction of the main twist</td>
</tr>
<tr>
<td>4</td>
<td>13.408</td>
<td>Z-axis direction of the main fluctuations</td>
</tr>
<tr>
<td>5</td>
<td>17.19</td>
<td>X-axis and Y-axis direction of the main fluctuations</td>
</tr>
<tr>
<td>6</td>
<td>19.252</td>
<td>X-axis torsion and Y-axis direction of the main fluctuations</td>
</tr>
<tr>
<td>7</td>
<td>20.492</td>
<td>Z-axis up and down the direction of the main fluctuations</td>
</tr>
<tr>
<td>8</td>
<td>24.782</td>
<td>X-axis direction of the main twist</td>
</tr>
<tr>
<td>9</td>
<td>29.369</td>
<td>X-axis torsion and Y-axis direction of the main fluctuations</td>
</tr>
<tr>
<td>10</td>
<td>30.384</td>
<td>X-axis and Y-axis direction of the main fluctuations</td>
</tr>
</tbody>
</table>

Several representative vibration modes are shown as follows:

a) The 1st order vibration mode  
b) The 4th order vibration mode
c) The 7th order vibration mode  
d) The 10th order vibration mode

**Figure 1.** The vibration mode shapes.

Combined with the structure of the natural frequency and vibration diagram can be drawn: with the increase of vibration order, the natural frequency also increases accordingly. The vibration modes of the bridge are mainly deformed in the vertical direction, indicating that the vertical stiffness of the bridge is small.

### 5.3 Time History Analysis

The model is calculated by time history analysis, the time history responses of the stress and displacement on the bridge are obtained. The time history analysis of the seismic wave selected the Tianjin wave, Analysis time step is 0.01s, Calculate duration is 13. The structural damping coefficient is 0.05, and the time history analysis of the bridge is carried out.

**Table 2.** Position and node number of data points

<table>
<thead>
<tr>
<th>Date point</th>
<th>Position</th>
<th>Node number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The nodes near the bottom of the side span T-shaped beam</td>
<td>33429</td>
</tr>
<tr>
<td>2</td>
<td>The nodes near the boundary of the web and flange of the side span T-shaped beam</td>
<td>36039</td>
</tr>
<tr>
<td>3</td>
<td>The nodes near the bottom of the 1/4 side span T-shaped beam</td>
<td>35138</td>
</tr>
<tr>
<td>4</td>
<td>The nodes near the boundary of the web and flange of the 1/4 side span T-shaped beam</td>
<td>72163</td>
</tr>
<tr>
<td>5</td>
<td>The nodes near the crossing the bottom of the T-shaped beam</td>
<td>27647</td>
</tr>
<tr>
<td>6</td>
<td>The nodes near the boundary of the web and flange of the crossing T-shaped beam</td>
<td>82310</td>
</tr>
</tbody>
</table>

Displacement response and stress response diagram are as follows:

a) UX-6: The x-displacement response of data point 6  
b) UY-6: The y-displacement response of data point 6
c) UZ-6: The z-displacement response of data point 6

**Figure 2.** Displacement response of data points 6 (displacement unit: m)

a) SY-1: y-stress response of data points 1                  b) SY-4: y-stress response of data points 4

**Figure 3.** The Stress response of data points 1 and 4 (Stress unit: Pa)

<table>
<thead>
<tr>
<th>Data point</th>
<th>Node number</th>
<th>Displacement</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X  Y  Z</td>
<td>X  Y  Z</td>
</tr>
<tr>
<td>1</td>
<td>33429</td>
<td>1.30 0.09 0.03</td>
<td>0.06 4.36 0.43</td>
</tr>
<tr>
<td>2</td>
<td>36039</td>
<td>0.30 0.01 0.36</td>
<td>0.07 0.40 0.08</td>
</tr>
<tr>
<td>3</td>
<td>35138</td>
<td>0.60 3.25 1.11</td>
<td>0.46 0.03 0.05</td>
</tr>
<tr>
<td>4</td>
<td>72163</td>
<td>0.49 3.82 1.44</td>
<td>0.08 0.11 0.03</td>
</tr>
<tr>
<td>5</td>
<td>27647</td>
<td>0.06 3.8 0.05</td>
<td>2.20 0.03 0.18</td>
</tr>
<tr>
<td>6</td>
<td>82310</td>
<td>0.07 3.64 1.12</td>
<td>0.45 0.02 0.03</td>
</tr>
</tbody>
</table>

### 6. CONCLUSIONS

The displacement of the bridge in the vertical direction is much larger than those in the other two directions, the closer to the middle position, the greater the vertical displacement. In general, the displacement at the bottom of the bridge is larger than that of the upper part, and the stress at the end of the bridge is the largest, and the closer to the middle of the bridge, the smaller the stress is. And for the reinforced concrete T-shaped beam bridge, the overall structure is better than the local results, for the multiple seismic zone, as far as possible to maintain the integrity of the structure.

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