Reasonable Location and Shape Optimization of Air Cushion Surge Chamber

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Abstract

Reasonable location of air cushion surge chamber (ACSC) is the starting point of overall optimum design of long water delivery system, but it is mainly considered on the geological and topographical conditions and little involved in the regulation guarantee of unit. Based on the theory of 1-D transient flow and method of characteristics(MOC), a mathematical model of the transition process of the long water delivery system with ACSC is established. The reasonable location and shape optimization of ACSC in water delivery system are studied. The results show that for the ACSC, which’s shape parameters has already been determined, has the optimal location in the water delivery system to make the water hammer pressure and surge pressure at the end of the spiral case approximately equal. When the surge pressure is in control, it can achieve good protective effect through the optimization of throttled orifice. So, the ACSC can be next to the unit with the better protective effect as close as possible.

Keywords: air cushion surge chamber, reasonable location, throttled orifice, maximum surge

1. INTRODUCTION

In order to reduce the water hammer pressure caused by unit’s load rejection, design code for surge chamber of hydropower station pointed out that the surge chamber location should be next to the unit as close as possible to reduce the pressure standard of the entire waterway system and save the cost of project. For the long water delivery system with air cushion surge chamber (ACSC), the pressure of the whole channel system in the process of hydraulic transition can be divided into two parts (Yang et al., 2007). The one was water hammer pressure produced in the process of the closure of units, the other was surge pressure caused by the rising of ACSC water level (also called air chamber overpressure). The former is caused by the elasticity of water, mainly related to the characteristics of the water in the pressure pipe after the ACSC. The latter is caused by the inertia of water, mainly related to the water characteristics of the water conveyance tunnel in front of the ACSC. Two pressure do not happened at the same time, and the surge pressure will rise further after the water hammer pressure disappeared. The maximum value is likely to exceed the water hammer pressure caused by the closure of units. ACSC is closer to the unit, the inertia of water in water conveyance tunnel is larger, and the elasticity of pressure pipeline is smaller. The water hammer pressure will be smaller and the surge pressure will be larger. If the ACSC is too close to the plant, it may cause the distribution of the pressure of the whole channel system uplift, leading to the opposite effect. Reasonable location of ACSC should be able to make the pressure of water hammer pressure and surge pressure approximately equal (Yao et al., 2007; Parmakian, 1963; Cheng et al., 2007; Bao et al., 2007; Wan et al., 2013; Ike et al., n2013; Liu 1995; Popoola et al., 2016; Rajput et al., 2016; Svee 1972). The pressure of the ACSC is related to the characteristics of the water in the tunnel, the cross area of the air chamber and the throttled orifice at the bottom of the ACSC. And the throttled orifice is related to the reflection effect of the water hammer as well. Despite of the geological and topographical conditions, the reasonable location of the ACSC should be taken the regulation guarantee parameters of the water conveyance tunnel, the units and the parameters of the ACSC into consideration.

In this paper, a mathematical model is established for the transition process of a long water delivery system with ACSC, and the parameters and the location of ACSC in the system are studied. Corresponding results can provide a reference for engineering design.
2. MATHEMATICAL MODEL

2.1 Basic equation of the pipeline with pressure flow

Basic equations of the pipeline with pressure flow are as following (Wylie and Streeter 1993):

\[
\frac{Q}{A} \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} - \frac{Q}{A} \sin \beta = 0 \quad (1)
\]

\[
\frac{Q}{A^2} \frac{\partial^2 Q}{\partial x^2} + \frac{1}{A} \frac{\partial Q}{\partial t} + \frac{fQ|Q|}{2DA^2} = 0 \quad (2)
\]

Where \( H \) is the piezometer tube water head, \( m \); \( Q \) is the discharge in the pipe, \( m^3/s \); \( A \) is the area of the pipe, \( m^2 \); \( t \) is a variable time, \( s \); \( a \) is velocity of water hammer wave, \( m/s \); \( g \) is the acceleration of gravity; \( x \) is the distance along the tube axis, \( m \); \( f \) is the friction coefficient; \( \beta \) is the angle between tube axis and horizontal plane. The above formula can be simplified to the standard hyperbolic partial differential equations, so the method of characteristics can be used in transforming the pipeline water hammer into the same solution of the characteristic compatibility equation.

2.2 Mathematical model of method of characteristics

The method of characteristics is used to convert partial differential equations of water hammer for compatibility equation, and the turbine characteristic curve of the positive and negative compatibility equations are:

\[
C^+ : H_{pi} = C_p - B_p Q_{pi} \quad (3)
\]

\[
C^- : H_{pi} = C_M + B_M Q_{pi} \quad (4)
\]

Where, \( H_{pi} \) is the pressure of point P on the cross section i; \( C_p, B_p, C_M, B_M \) is the known quantity during \( t-\Delta t \). \( Q_{pi} \) is the discharge of point P on the cross section i.

Assuming that the upper and lower boundary node number of turbine are 1 and 2 respectively, and the head balance equation can be got based on the formula (3)-(4):

\[
h = \frac{(C_{p1} - C_{M2})}{H_r} - \frac{(R_{p1} + R_{M2})Q_r}{H_r} \quad (5)
\]

Where, \( H_r, Q_r \) are the rated head and discharge under stable condition. \( C_{p1}, R_{p1}, C_{M2}, R_{M2}, q \) is constant under stable condition.

2.3 Mathematical model of ACSC

A mathematical model of ACSC is established, as shown in Figure 1.

![Figure 1 Schematic diagram of the mathematical model of ACSC](image-url)
For the mathematical model of the ACSC (shown in Figure 1), the basic equation of the ACSC is as follows (Wylie, 1993):

**Continuity Equation**

\[ Q_{p1} = A_{st} \frac{dz_{st}}{dt} + Q_{p2} \]  

(6)

**Motion Equation**

\[ H_p = z_{st} + \frac{p - P_0}{\gamma} + kQ_{st} |Q_{st}| \]  

(7)

**Compatibility equation**

\[ H_p = C_{p1} - B_{p1}Q_{p1} \]
\[ H_p = C_{m2} + B_{m2}Q_{p2} \]  

(8)

**Polytropic gas equation**

\[ pV^n = C \]  

(9)

Where \( P \) is the absolute gas pressure in the air chamber, \( \text{pa} \); \( V \) is the air volume in the air chamber, \( \text{m}^3 \); \( m \) is the polytrophic exponent of ideal gas, its value is 1.0 isothermally and is 1.4 adiabatically; \( H_p \) is the pressure in the connection joint of ACSC and pipe, \( \text{m} \); \( Q_{p1} \) and \( Q_{p2} \) is the discharge on both sides of the ACSC, \( \text{m}^3/\text{s} \); \( z_{st} \) is the water level in ACSC, \( \text{m} \); \( z_0 \) is the top height of ACSC, \( \text{m} \); \( A_{st} \) is the cross-sectional area of the air chamber, \( \text{m}^2 \); \( k \) is the hydraulic loss coefficient in the connection node of ACSC and pipeline.

Simultaneous equations (6) ~ (9) can be obtained \( H_p, z_{st}, Q_{p1}, Q_{p2}, Q_{st}, p \).

**3. THE EFFECT OF SHAPE PARAMETERS ON THE PRESSURE AT THE END OF THE SPIRAL CASE**

In the long water delivery system of the hydropower station, maximum pressure at the end of the spiral case is controlled by the larger value of water hammer pressure and surge pressure. Among them, the water hammer pressure generally appeared very fast, and damping is also fast. Surge pressure generally occurs for a long time, and damping is slow. The surge and pressure in air chamber of ACSC is related to the hydraulic inertia of system as well as shape parameters ACSC.

**3.1 The influence of the area of throttled orifice**

The area of the throttled orifice not only affects the effect of the water hammer wave of the ACSC, but also determines the discharge in and out of the ACSC (Yang, 2000). When the area of the throttled orifice is too small, it will be bad to the reflection of water hammer wave and lead to the increase of water hammer pressure. But the small area of the throttled orifice will restrain water in the pipe flow into the surge chamber, and can reduce the highest surge of ACSC and the pressure in air chamber. So the maximum pressure at the end of the spiral case is small. When the area of the throttled orifice is larger, the reflection of water hammer wave will be effective, and the pressure of water hammer is small. But the larger area of the throttled orifice will lead to more water flow into the ACSC and increase the highest surge and pressure in ACSC. And it will cause the larger pressure at the end of the spiral case, which even exceed the water hammer pressure and become the control values. Figure 2 is the maximum pressure trend line at the end of the spiral case under different diameter of throttled orifice. In the graph, it can be seen that the increase of diameter of throttled orifice, falling water hammer
pressure and rising surge pressure. Maximum pressure at the end of the spiral case is decided by the larger values of two pressures. When the diameter of throttled orifice is more than 2.59 m, the surge pressure exceed the water hammer pressure and become the control value of the maximum pressure at the end of the spiral case. For ACSC, there is an optimal area of throttled orifice, which makes the maximum pressure at the end of the spiral case minimum.

![Figure 2](image1.png)

**Figure 2** The trend line of maximum pressure at the end of the spiral case with different area of throttled orifice

![Figure 3](image2.png)

**Figure 3** The trend line of maximum pressure at the end of the spiral case with different area of air chamber

### 3.2 The influence of the area of air chamber

The area of throttled orifice influenced the discharge in and out of ACSC, and the area of air chamber affected the surge level of ACSC. When the discharge in and out of ACSC were determined, smaller area of air chamber will result in higher surge, leading to great pressure in the air chamber (Peng, 2000). Figure 3 is the trend line of maximum pressure at the end of the spiral case with different area of air chamber. It can be seen from the chart, along with the increase of diameter of air chamber, maximum pressure of water hammer pressure and surge pressure were decreased. Effects on surge pressure caused by the changes of diameter of air chamber is more obvious, because the surge pressure is influenced by maximum surge and maximum chamber pressure in ACSC. When the diameter of air chamber is increased, the maximum surge of ACSC decreased while the maximum pressure in air chamber is also reduced. The maximum surge pressure decreased faster under the two factors. When the diameter of air chamber is more than 19.26m, the control value of maximum pressure at the end of the spiral case changed from the surge pressure to the water hammer pressure. Therefore, for ACSC, there also exist the optimal area of air chamber making maximum pressure at the end of the spiral case is minimum.

### 4. REASONABLE LOCATION OF ACSC

There exists the optimal area of throttled orifice and air chamber in ACSC. When the basic parameters of the ACSC were determined, the influence of the location of the ACSC and the unit on the maximum pressure at the end of the spiral case is very important. In view of a practical engineering system, the overall mathematical model is established, and the reasonable location of the ACSC is studied.

#### 4.1 Engineering situation
A layout diagram of the system with ACSC for hydropower station is as shown in Figure 4. There were two units in the system, which’s capacity of unit is 70.0 MW, design head of unit is 199.2 m, design discharge of unit is 39.4m3/s, length of water delivery tunnel is 2822.75m, diameter of water delivery tunnel is 5.25 m, length of the pressure pipe is 230.0m, area of air chamber is 291.2m2, diameter of throttled orifice is 2.6m, initial height of air chamber is 16.0 m, initial absolute pressure head of air chamber is 101.09 m, polytropic exponent of ideal gas is 1.4. The calculation conditions was that the upper reservoir is the normal storage level with rated head, and two units reject load at the same time. In the calculation process, the guide vane of the unit is closed in 11s with a linear closure law.

**Figure 4** Layout of water delivery system

### 4.2 Calculation results and theoretical analysis

#### 4.2.1 Parameters of ACSC remain unchanged

In the calculation process, the parameters and initial gas chamber absolute pressure of air cushion surge pressure chamber remain unchanged. The location of ACSC in water conveyance system is adjusted through the change of pressure pipeline length L. Calculation results are shown in Table 1 and in Figure 5-6.

**Table 1** Calculation results with different length L

<table>
<thead>
<tr>
<th>length L</th>
<th>Initial water level</th>
<th>The maximum surge</th>
<th>Maximum pressure in air chamber</th>
<th>Maximum water hammer pressure</th>
<th>Maximum surge pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1607.39</td>
<td>1611.433</td>
<td>151.985</td>
<td>249.911</td>
<td>263.468</td>
</tr>
<tr>
<td>130</td>
<td>1607.57</td>
<td>1611.539</td>
<td>150.676</td>
<td>253.271</td>
<td>262.424</td>
</tr>
<tr>
<td>230</td>
<td>1607.75</td>
<td>1611.643</td>
<td>149.364</td>
<td>256.692</td>
<td>261.243</td>
</tr>
<tr>
<td>530</td>
<td>1608.29</td>
<td>1611.947</td>
<td>145.380</td>
<td>267.742</td>
<td>258.877</td>
</tr>
</tbody>
</table>

**Figure 5** The trend line of pressure at the end of spiral case changing with the time
Table 1 is calculation results for different length of pressure pipeline, Figure 5 is the trend line of pressure at the end of spiral case changing with the time and Figure 6 is the trend line of absolute pressure head of air chamber changing with the time. Table 1 and figure 5 shows that when the length of water delivery tunnel becomes shorter, the head loss in tunnels decreases. In order to keep the initial pressure in air chamber equal, the initial water level in air chamber rise. And when the length of water delivery tunnel becomes shorter, the inertia in tunnels is reduced gradually, leading to absolute value of rising surge decreases. Under the interaction of the two factors, although the maximum surge in ACSC continues to rise, the maximum pressure head in air chamber was reduced. Figure 6 is the trend line of absolute pressure head of air chamber changing with the time. From the figure it can be seen that when the pressure pipeline length is short, the water hammer pressure was small, surge pressure was large, and the pressure at the end of spiral case is controlled by the surge pressure. When the length of pressure pipeline increased, the control value of pressure at the end of spiral case has gradually changed from surge pressure to water hammer pressure. This is because when the length of the pressure pipeline is shorter, the distance between ACSC and the unit is relatively close. Reflection of water hammer wave is very good, and the water hammer pressure rise value is small. But at this time the length of water delivery tunnel is longer, the inertia of water in tunnel is large, resulting in higher surge and larger pressure in the air chamber, and the surge pressure becomes the control value of maximum pressure at the end of spiral case. When the length of pressure pipeline is 230.0m, the value between water hammer pressure and surge pressure was minimum, and pressure at the end of spiral case was minimum too.

4.2.2 Shape parameter optimization

For different locations of the ACSC, there is an optimum area of throttled orifice and air chamber. When the distance between ACSC and unit is close, surge pressure becomes the control pressure. In order to reduce the surge pressure, it can be achieved by increasing the area of air chamber or decreasing the area of throttled orifice. Increasing the area of air chamber is not economical. When the distance between ACSC and unit is far, water hammer pressure becomes the control pressure. In order to reduce the water hammer pressure, it is unreasonable to reduce the area of air chamber, which will lead to greater water hammer pressure (Zhao et al., 2004; You et al., 2003; Zhang et al., 2003). So it only can optimize the area of throttled orifice to regulate the water hammer pressure to keep the water hammer pressure and surge pressure at the end of the spiral case approximately equal. Therefore, the area of air chamber and absolute pressure head and height of initial air chamber keep unchanged, and the area of throttled orifice under different locations is optimized. The specific calculation results are shown in Table 2 and Figure 7.

Table 2 Parameter optimization under different locations

<table>
<thead>
<tr>
<th>length L</th>
<th>Diameter of throttled orifice</th>
<th>The maximum surge</th>
<th>Maximum pressure in air chamber</th>
<th>Maximum water hammer pressure</th>
<th>Maximum surge pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2.34</td>
<td>1611.128</td>
<td>146.720</td>
<td>258.121</td>
<td>257.904</td>
</tr>
<tr>
<td>130</td>
<td>2.42</td>
<td>1611.339</td>
<td>147.237</td>
<td>258.677</td>
<td>258.824</td>
</tr>
<tr>
<td>230</td>
<td>2.50</td>
<td>1611.538</td>
<td>147.564</td>
<td>259.528</td>
<td>259.372</td>
</tr>
<tr>
<td>530</td>
<td>2.82</td>
<td>1612.135</td>
<td>148.543</td>
<td>263.132</td>
<td>263.098</td>
</tr>
</tbody>
</table>
It can be seen from table 2 and Figure 7, when the surge pressure and water hammer pressure is approximately equal, with the increase of the length of the pressure pipeline, the optimal diameter of throttled orifice continue to increase. The distance between ACSC and unit is more far, the reflection effect of the water hammer wave is smaller, and the larger area of throttled orifice is needed in order to achieve a more ideal effect. Increase of the area of throttled orifice lead to more water flow into the ACSC, increasing maximum pressure of the air chamber with the location of the ACSC moving. When the length of the pressure pipeline is 30m, the maximum pressure of the air chamber and the maximum pressure at the end of the spiral case are 146.72m and 258.12m respectively, which are less than the condition of length of the pressure pipeline is 230.0m. With the increase of the length of pressure pipeline, the maximum pressure of air chamber and the maximum pressure at the end of the spiral case present an upward trend, which indicates there isn’t the optimal location. Therefore, through the optimization of the area of throttled orifice, the ACSC can be next to the unit as close as possible in the case of better protective effect.

4.2.3 Influence of the initial gas height

The Polytropic gas equation (9) shows that when the change of temperature did not consider, \( C=PV^n \) remain unchanged in the process of transition. Therefore, \( C \) can be regard as equivalent volume value of gas. Obviously, when the cross sectional area of surge chamber is unchanged, the greater volume of gas chamber, the better reflection of ACSC to relieve water hammer in the system. The specific calculation results under different initial gas height are shown in Table 3 and Figure 8.

<table>
<thead>
<tr>
<th>Initial gas height/ m</th>
<th>The maximum surge/ m</th>
<th>Maximum pressure in air chamber/ m</th>
<th>Maximum water hammer pressure/ m</th>
<th>Maximum surge pressure/ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.0</td>
<td>1610.617</td>
<td>168.153</td>
<td>265.894</td>
<td>279.003</td>
</tr>
<tr>
<td>18.0</td>
<td>1610.892</td>
<td>159.132</td>
<td>262.611</td>
<td>270.288</td>
</tr>
<tr>
<td>16.0</td>
<td>1611.129</td>
<td>152.417</td>
<td>260.340</td>
<td>263.817</td>
</tr>
<tr>
<td>14.0</td>
<td>1611.339</td>
<td>147.237</td>
<td>258.677</td>
<td>258.824</td>
</tr>
<tr>
<td>12.0</td>
<td>1611.531</td>
<td>143.159</td>
<td>257.404</td>
<td>254.921</td>
</tr>
<tr>
<td>10.0</td>
<td>1611.706</td>
<td>139.827</td>
<td>256.402</td>
<td>251.784</td>
</tr>
</tbody>
</table>
Figure 8 The trend line of pressure at the end of spiral case changing with the initial air height

It can be seen from table 3 and Figure 8, with the increase of the length of the initial air height, the pressure at the end of spiral case and the air chamber shown a diminishing trend, and the maximum water hammer pressure reduced faster. When the initial gas height is small, maximum water hammer pressure is the control pressure, and the ACSC should close to the unit; When the initial gas height is large, maximum surge pressure is the control pressure, and the ACSC should far away from the unit. Although the larger initial gas height has advantage to reduce the water hammer pressure, but obviously it is not the optimal solution. The optimal initial air chamber height should ensure that the maximum water hammer pressure is roughly equal to the maximum surge pressure.

5. CONCLUSION

Combined with theoretical analysis and the numerical model of actual water engineering, the reasonable location of air cushion surge chamber in the water delivery system was studied. The results show that the optimal location can be obtained in the water delivery system for the air cushion surge chamber which’s shape parameter was definitized. And the optimal protective effect can be obtained when the air cushion surge chamber is in the optimal location. When the maximum surge pressure is the control pressure at the end of the spiral case, Through the optimization of the area of throttled orifice, it can achieve ideal protective effect. Therefore the air cushion surge chamber can be as close as possible to the unit.

REFERENCES


