A Multi-wing Butterfly Chaotic System and Its Circuit Design based on Sawtooth Wave

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Abstract
Based on the sawtooth wave function, a piecewise linear function is designed in this paper. Then, based on a two-wing butterfly chaotic system, Through replacing the state variable x of the two-wing butterfly chaotic system with the piecewise linear function, and modifying the system parameters appropriately, a novel multi-wing butterfly chaotic system is presented. Through numerical simulation, it is analyzed for the phase diagrams of the original two-wing butterfly chaotic system and the novel multi-wing butterfly chaotic system. Then it is verified for chaotic characteristic of the new multi-wing butterfly chaotic system via analyzing the Lyapunov exponent spectrum and the bifurcation diagram. At the end, an electronic circuit is designed to implement the system, and the circuit simulation experiment is carried out. The circuit simulation results are in consistent with the numerical simulation results, which verify the realizability of the system.

Keywords: Chaos, Sawtooth wave function, Multi-wing butterfly chaotic system, Circuit design

1. INTRODUCTION

In recent years, along with the people’s further study on the chaotic phenomena of the nonlinear systems and the chaotic application, chaos has received increased attention in some application fields, such as biological system, electrical system, and secret communication field, etc.

At present, the multi-scroll chaotic system, the two- and four-wing butterfly chaotic system have been studied in a number of literatures, while, people pay little attention to multi-wing butterfly chaotic system [12-18]. It is the common method for constructing the multi-wing butterfly chaotic systems to design the appropriate nonlinear functions based on the two-wing butterfly chaotic systems. Reference designed the piecewise nonlinear hyperbolic tangent function, and presented the grid multi-wing butterfly chaotic systems with the nonlinear function. Based on three-dimension Lorenz system, reference constructed a multi-wing butterfly chaotic system via adding a nonlinear function and a first-order differential equation. Reference designed a new nonlinear function based on the step function, and constructed a multi-wing butterfly chaotic system via replacing the state variables of the Liu system with the nonlinear function. Reference presented a grid multi-wing butterfly chaotic system based on two piecewise nonlinear hysteresis functions. However, the piecewise linear function of these documents are very complicated. It is very difficult for circuit implementation. The sawtooth wave is a common function, and it is easy for circuit implementation. Therefore, it is significance to study multi-wing butterfly chaotic system based on the sawtooth wave function.

In the following section, we research a five-term simple two-wing butterfly chaotic system. In the section 3, we present a multi-wing butterfly chaotic system based on the two-wing butterfly chaotic system, and the phase diagram, the Lyapunov exponent spectrum and the bifurcation diagram of the new multi-wing butterfly chaotic system are studied through numerical simulation. In the section 4, an electronic circuit is designed to implement the system. Finally, the results and conclusions are drawn in section 5.

2. BASIC MODEL SIMULATION

Reference presented a five-term simple chaotic system as

\[
\begin{align*}
&\dot{x} = a(y - x), \\
&\dot{y} = -xz, \\
&\dot{z} = -b + xy, \\
\end{align*}
\]

(1)

where \(a\) and \(b\) are system parameter. When \(a = 5\) and \(b = 90\), the system (1) can generate a two-wing butterfly chaotic attractors, as shown in figure 1.
3. IMPROVED MODEL SIMULATION

3.1. Phase Diagrams

Based on the sawtooth wave function, a new piecewise linear function is designed as

$$ f(x) = x - \sum_{n=0}^{N} \left[ \text{sgn}(x - 2n - 1) + \text{sgn}(x + 2n + 1) \right] $$

(2)

where $N \in \{0, 1, 2, \ldots\}$.

Through replacing the state variable $x$ of the system (1) with the piecewise linear function $f(x)$, and modifying the system parameters appropriately, a novel system is obtained as

$$
\begin{align*}
\dot{x} &= a(y - f(x)), \\
\dot{y} &= -c f(x) z, \\
\dot{z} &= -b + c f(x) y.
\end{align*}
$$

(3)

Where $a$, $b$ and $c$ are system parameter. When $a = 5$, $b = 4.5$, $c = 20$, the system (3) can generate $(4N+6)$-wing butterfly chaotic attractors. The six- and ten-wing butterfly chaotic attractors is shown in figure 2.
3.2. Basic dynamic behavior

The Lyapunov exponent spectrum of the system (3) and its bifurcation diagram which varies with the coefficient $a$ are shown in figure 3. From the figure 3, we can see that, when $a \in [0,4.20] \cup [5.96,6]$, the largest Lyapunov exponent of the system (3) approximate equal to zero ($LE_1 \approx 0$ and $LE_3 < LE_2 < 0$), so it is in the periodic state; when $a \in (4.20,5.96)$, the largest Lyapunov exponent of the system (3) is greater than zero ($LE_1 > 0$, $LE_2 \approx 0$, and $LE_3 < 0$), so it is in the chaotic state. The bifurcation diagram is consistent with the corresponding Lyapunov exponent.
4. EXPERIMENTAL ANALYSIS

In the circuit design, all the multipliers are of type AD633JN, and their gain is 0.1. All operational amplifiers are selected as UA741CN. Their supply voltage $E = \pm 15V$, and their saturated voltage is $V_{sat} \approx \pm 13.5V$.

According to the system (3), the circuit of multi-wing butterfly chaotic attractors is designed, as shown in Figure 4.

Let $N = 1$, the piecewise linear function (2) is changed as follow

$$f(x) = x - \sum_{n=0}^{i} \left[ \text{sgn}(x - 2n - 1) + \text{sgn}(x + 2n + 1) \right].$$

The circuit diagram of the piecewise linear function (4) is shown in Figure 5.
According to the figures 4 and 5, the circuit equation can be obtained as

\[
\begin{align*}
\frac{dx}{d\tau} &= \frac{1}{R_0 C_0} \left( \frac{R}{R_1} y - \frac{R}{R_2} f(x) \right), \\
\frac{dy}{d\tau} &= \frac{1}{R_0 C_0} \left( -\frac{R}{10R_3} f(x) z \right), \\
\frac{dz}{d\tau} &= \frac{1}{R_0 C_0} \left( \frac{R_k}{R_1 + R_k + R_0} E - \frac{R}{10R_4} f(x) y \right).
\end{align*}
\]

(5)

\[
\begin{align*}
f(x) &= x - V_{sat} \frac{R}{R_{v1}} \text{sgn} \left( x - \frac{R_1 + R_2}{R_1 + R_2 + R_3} E \right) + V_{sat} \frac{R}{R_{v1}} \text{sgn} \left( x - \frac{R_1 + R_2}{R_1 + R_2 + R_3} E \right) \\
&\quad + V_{sat} \frac{R}{R_{v1}} \text{sgn} \left( x - \frac{R_5}{R_1 + R_4} E \right) + V_{sat} \frac{R}{R_{v1}} \text{sgn} \left( x - \frac{R_5}{R_1 + R_4} E \right) + V_{sat} \frac{R}{R_{v1}} \text{sgn} \left( x - \frac{R_5}{R_1 + R_4} E \right).
\end{align*}
\]

(6)

To observe the output wave experimentally, the (5) must take timescale transformation, that is, letting \( \tau = \tau_0 \), \( \tau_0 = 10^{-3} \), the (5) can be changed as

\[
\begin{align*}
\frac{dx}{dt} &= 10^{-3} \frac{R}{R_1} y - \frac{R_k}{R_1} f(x), \\
\frac{dy}{dt} &= 10^{-3} \frac{R}{10R_3} f(x) z, \\
\frac{dz}{dt} &= 10^{-7} \frac{R_k}{R_1 + R_k + R_0} E - \frac{R}{10R_4} f(x) y.
\end{align*}
\]

(7)

Letting \( R = 10k\Omega \), \( R_0 = 10k\Omega \), and \( C_0 = 10nF \), according to the (2), (3), (6) and (7), we can obtain \( R_1 = R_2 = 20k\Omega \), \( R_3 = R_4 = 500\Omega \), \( R_5 = 291k\Omega \), \( R_6 = 9k\Omega \), \( R_7 = 2k\Omega \), \( R_8 = 1k\Omega \), \( R_9 = 12k\Omega \), and \( R_{v1} = 135k\Omega \).

According to the circuit diagrams in figures 4 and 5, the multi-wing butterfly chaotic attractors are obtained via circuit simulation software Multisim 10.0, as shown in figure 6. When the \( k \) is switched off, the circuit generates six-wing butterfly chaotic attractors, as shown in figure 6(a). When the \( k \) is switched on, the circuit creates ten-wing butterfly chaotic attractors, as shown in figure 6(b).

From the figures 2 and 6, we can see that the circuit experimental results are in consistent with numerical simulation results, which verify the realizability of the system.
5. RESULTS AND CONCLUSIONS

This paper, a novel multi-wing butterfly chaotic system is presented via designing a new piecewise linear function based on the sawtooth wave function. Through the numerical simulation and circuit simulation, the six- and ten-wing butterfly chaotic attractors are obtained. As the sawtooth wave function is easy for circuit implementation, it is potential application value to construct multi-wing butterfly chaotic system based on the sawtooth wave function.

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References


