Characterization and development of closed form solutions for axle load spectra associated with trucks circulating the highways of Colombia

Caracterización y desarrollo de ecuaciones analíticas para los espectros de carga por eje asociados con los camiones que circulan las carreteras de Colombia

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ABSTRACT: An appropriate design of pavement structures requires estimation of the effects produced by traffic loads. For this reason, it is necessary to predict the number of vehicles traveling on road surfaces, identifying the axle configurations and the magnitude of the load. Thirty-eight (38) mobile weigh stations were positioned on different strategic points of the Colombian national road network to characterize the traffic condition of the country, 59,622 trucks were observed. This paper presents Axle Load Spectra that could be used to characterize the traffic condition of the Colombian’s highway network. Data showed that the different axle configurations (Single, Tandem and Tridem) exhibit two peaks, associated with loaded and unloaded truck conditions. It was noted that a significant number of axles exceeds the permitted load limit, which could explain the critical state of the road network in Colombia. Finally, a mixture of three (3) theoretical distributions was used to characterize the overall axle load distributions. The proposed model presents acceptable predictive capabilities; therefore, it has potential to be used as a tool in the design process of the pavement.

RESUMEN: El diseño adecuado de las estructuras de pavimento requiere de la estimación de los efectos producidos por las cargas de tráfico. Por esta razón, es necesario predecir el número de vehículos que se desplazan sobre superficies de carretera e identificar las configuraciones de ejes y la magnitud de la carga. Treinta y ocho (38) operativos de pesaje móviles fueron colocados en diferentes puntos estratégicos de la red vial nacional colombiana con el objetivo de caracterizar la condición de tráfico del país; en estos operativos se evaluaron 59,622 camiones. Este artículo presenta espectros de carga por eje que podrían ser utilizados para caracterizar el estado del tráfico de la red de carreteras de Colombia con el fin de dimensionar las estructuras de pavimentos. Los datos mostraron que las diferentes configuraciones de ejes (simple, doble y Tridem) muestran dos picos, asociados a las condiciones de camiones cargados y descargados. Se observó que un número significativo de ejes excede el límite de carga permitido, lo que podría explicar el estado crítico de la red de carreteras en Colombia. Por último, una mezcla de tres (3) distribuciones teóricas se utilizó para caracterizar las distribuciones generales de carga por eje. Se observó que los modelos propuestos presentan capacidades de predicción aceptables, por lo tanto, tienen potencial para ser utilizados como una herramienta confiable en el proceso de diseño de pavimentos.

1. Introduction

Traffic characterization is necessary to properly define the geometric and structural characteristics of road infrastructure. Historically in Colombia, in the field of Pavement Engineering, designers have chosen the Equivalent Single Axle Load (ESAL) concept to characterize traffic conditions of roads. The ESAL concept provides an effective method to convert the damage caused by repetitions of different axle configurations carrying different...
levels of loads into a standard variable [1]; however, this methodology does not provide information to assess whether the vehicles circulate with overloads.

The characterization of the traffic through load spectra has been developed in response to the limitations of the traditional approach [2, 3]. The load spectra represent the percentage of the total axle applications within different load intervals for single, tandem, and tridem axles. Furthermore, it is important to note that the Mechanistic-Empirical Pavement Design Guide (MEPDG) defined the use of axle load spectra to represent the vehicle loads in pavement design [2, 3].

Many researchers have statistically modeled axle load spectra using Log-normal and Normal distributions, and, combinations among them [4-7]. Some authors have proposed the use of a combination of two lognormal and two normal distributions respectively, to characterize axle load spectra [8]. In addition, it has been suggested that closed-form solutions using a mixture of two normal distributions could be used to represent axle load spectra [7]. On the other hand, it is important to highlight that the intermediate section of the distribution associated with the section between the peaks of the spectrum is not fully described by the use of two distributions.

Moreover, different investigations have proposed the implementation of axle load spectra in the pavement design process, using a practical approach to better define pavement thicknesses [9]. In addition, a study evaluated the implementation of load standard spectra for design of pavements in the state of Arkansas, finding that the axle load spectra for the state of Arkansas is significantly different than the default spectra proposed by the MEPDG [10]. Furthermore, it has been shown that the spatial and temporal variations in axle load spectra have a negligible impact on pavement thickness design [11]. More recently,

Table 1 Description of axles obtained from the mobile weighing operatives

<table>
<thead>
<tr>
<th>Departure</th>
<th>Destination</th>
<th>Location of the weighing site</th>
<th>Quantity by axle type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauya</td>
<td>La Pintada</td>
<td>Single directional axle</td>
<td>3,764</td>
</tr>
<tr>
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<td>Santafé</td>
<td>Standard single axle</td>
<td>1,847</td>
</tr>
<tr>
<td></td>
<td>Antioquia</td>
<td>Tandem axle</td>
<td>2,614</td>
</tr>
<tr>
<td></td>
<td>Antioquia</td>
<td>Tridem axle</td>
<td>1,240</td>
</tr>
<tr>
<td>Barbosa</td>
<td>Arcabuco</td>
<td>Single directional axle</td>
<td>1,722</td>
</tr>
<tr>
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<td>Arcabuco</td>
<td>Standard single axle</td>
<td>602</td>
</tr>
<tr>
<td>Chiquinquirá</td>
<td>Tunja</td>
<td>Tandem axle</td>
<td>1,288</td>
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<tr>
<td>QUITAMA</td>
<td>La Palma</td>
<td>Tridem axle</td>
<td>929</td>
</tr>
<tr>
<td>Olanche</td>
<td>Chiquinquirá</td>
<td>Standard single axle</td>
<td>201</td>
</tr>
<tr>
<td>Sogamoso</td>
<td>El Crucero</td>
<td>Tandem axle</td>
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</tr>
<tr>
<td>Tuńja</td>
<td>Páez</td>
<td>Tridem axle</td>
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<tr>
<td>Cerritos</td>
<td>Cauya</td>
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<td>Chinchina</td>
<td>Manizales</td>
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<td>Caldas</td>
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<tr>
<td></td>
<td>Caldas</td>
<td>Tridem axle</td>
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<tr>
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<td>Higuerones</td>
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<td>524</td>
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<td>Popayén</td>
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<td>Peñol</td>
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<td>Zipaquirá</td>
<td>Briceno</td>
<td>Tridem axle</td>
<td>16</td>
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<tr>
<td>Portal</td>
<td>El Antojo</td>
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<td>Salitre</td>
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<td>Standard single axle</td>
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<td></td>
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<td>0</td>
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<td>S. Juan Villalobos</td>
<td>Pitalito</td>
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<td>788</td>
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<td>Puerto</td>
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<td>358</td>
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<td>Apia</td>
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<td>Honda</td>
<td>Dorada</td>
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<tr>
<td>Honda</td>
<td>Mariquita</td>
<td>Tridem axle</td>
<td>43</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>PROPORTION</td>
<td>38,186</td>
</tr>
</tbody>
</table>

Table 1 Description of axles obtained from the mobile weighing operatives
A methodology provided by the National Road Institute (INVIAS) was used to classify the goods transported on the Colombian roads as follows: agricultural products, animals or animal related products, forest products, mining and mineral products, manufactured products, industrial products, food, and others. The frequency of axles that violate the maximum load according to national standards \([19, 20]\) was determined. In addition, an analysis was performed to evaluate the types of goods transported in overload conditions; frequency distributions were developed by product type in order to identify economic sectors that contribute to the deterioration of roads throughout the country.

Axle load spectra tend to present multiple concentrations of central tendency, predominantly a bimodal shape. Theoretically, one could use a mixture of statistical distributions to characterize the overall axle load spectra \([7-9]\). Many investigations have shown that two or more normal (or lognormal) probability density functions can be added, with appropriate weight factors, to obtain the axle load spectra of the combined distribution as shown by Eq. \((1)\).

\[
    f^* (x) = \sum_{i=1}^{n} p_i f_i
\]

Where:
- \(f^*\): Combined axle load spectra,
- \(n\): Total number of distributions in the mixed model,
- \(p_i\): Proportion (weight) for each theoretical probability density function in the model.
- \(f_i\): Probability density function for each theoretical distribution.

Eq. \((1)\) provides a useful tool with an adequate theoretical and practical interpretation and reduces the number of parameters to be calibrated to characterize the traffic condition of a specific road. Figure 2 presents an overview of mixed models, as it is described by Eq. \((1)\).

Figure 1: Load spectra for the Colombian Caribbean region

Figure 2: Modeling of load spectra using bimodal normal distribution

Researchers have evaluated the effect of axle load spectra shifts on highway infrastructure damage and M&R costs \([12]\). The results showed that pavement damage is highly sensitive to changes in load spectra, highlighting the need for accurate load characterization. Likewise, it has been showed that increments on cracking, rutting, and roughness in flexible pavements are caused mainly by axle load distributions having heavier tails with infrequent extreme loads \([8]\).

In Colombia, several studies have been conducted to characterize load spectra and truck factors for the major roads of the country \([13-17]\). Following this line of research, this paper shows load spectra associated with different economic regions in Colombia, which are useful for estimating traffic loads on roads with similar traffic conditions.

2. Data collection

Thirty-eight (38) mobile weigh stations were positioned on different strategic points of the Colombian national road network to characterize the traffic condition of the country. Weighing operatives involved 17 departments and four of the five economic regions of Colombia. Each weighing operative was carried out for 7 days, with an intensity of 24 hours daily, with the goal of weighing in static condition all the axles of trucks circulating in one direction. Using appropriately calibrated scales, a sample of 59,622 trucks, corresponding to 94,135 axles of different configurations was obtained. Table 1 presents the details of the type of axle configurations observed by department.

3. Load Spectra Modeling

The construction of the load spectra was based on a quantitative and qualitative classification of the evaluated axles, gathering data from each weighing operative on the different economic zones, as proposed by the reference \([18]\). Figure 1 presents the load spectra of the tandem axle associated to the Colombian Caribbean region.
This general function of joint probability density has proven adequate in the studies made by researchers who have addressed the issue, however, has limitations, since it does not appropriately describe the full shape of axle spectra that present a wide separation between the main peaks [4]. In the present investigation, the use of a third statistical distribution is proposed to model the area between the main peaks, seeking to improve the model used to describe the general model of axle load spectra.

A mixed distribution model consisting of three statistical distributions was used to simulate the full form of the load spectra; two Log-normal probability functions \( f_1 \) and \( f_2 \) to capture the peaks of the spectrum and a Normal probability function \( f_3 \) to capture the intermediate distribution between the two peaks, as it is described in Eq. (2).

\[
f^* = p_1 \cdot f_1 + p_3 \cdot f_3 + p_2 \cdot f_2
\]

Where \( p_1 \), \( p_2 \) and \( p_3 \) represent the proportions (weights) of the probability functions \( f_1 \), \( f_2 \) and \( f_3 \) respectively. Note that \( p_3 \) proportion, corresponding to the intermediate distribution, can also be expressed as a function of the other distributions as presented in Eq. (3):

\[
f^* = p_1 \cdot f_1 + (1 - p_1 - p_2) + p_2 \cdot f_2
\]

Finally, the mixed distribution model used to simulate the full form of the load spectra is presented in Eq. (4).

\[
f^*(x) = \frac{p_1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{1}{2\sigma_1^2} (\ln x - \mu_1)^2} + \frac{(1 - p_1 - p_2)}{\sigma_2 \sqrt{2\pi}} e^{-\frac{1}{2\sigma_2^2} (x - \mu_2)^2} + \frac{p_2}{\sigma_3 \sqrt{2\pi}} e^{-\frac{1}{2\sigma_3^2} (\ln x - \mu_3)^2}
\]

This model also requires the estimation of the following parameters:

- \( \mu_1 \), \( \mu_2 \) and \( \mu_3 \): Means of the probability functions \( f_1 \), \( f_2 \) and \( f_3 \)
- \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \): Standard deviation for probability functions \( f_1 \), \( f_2 \) and \( f_3 \)

It is important to note that each of these parameters have a practical meaning. Standard deviations \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \) explain the variations in the axle weight of empty and fully loaded trucks, while \( \mu_1 \), \( \mu_2 \) and \( \mu_3 \) explain the average magnitude of the axle loads of trucks on the analyzed roads. The Levenberg-Marquardt technique was used to calibrate the general model [21]. The Levenberg-Marquardt technique represents an optimization algorithm that uses the weighted least squares technique to estimate the parameters presented on Eq. (4).

### 4. Results and analysis

A detailed description of the shape and the practical significance of the load spectra obtained for the single directional axles, single standard axles, tandem and tridem axles, having as reference the departments of the Colombian Andean region is presented. Figures 3, 4, 5 and 6 illustrate the axle load spectra associated with different departments of the Colombian Andean region.

**Figure 3 Load spectrum associated with the simple directional axles**

**Figure 4 Load spectrum associated with the standard axles**

**Figure 5 Load spectrum associated with the tandem axles**
4.3. Load spectra for tandem and tridem axles

Tandem and tridem axles have a similar shape to that of the standard single axle. However, the peaks shown in these load distributions are further apart from one another (see Figures 5 and 6), considering that tandem and tridem axles are more frequently found in trailer trucks with higher load capacity. Figures 5 and 6 illustrate a high frequency of axles travelling with overloads. This condition could be used to explain the extensive rutting damage found in Colombian roads. In addition, the reported low frequency of axles travelling with conditions of intermediate load can be associated with the low profitability and high costs associated with mobilizing freight trucks to transport goods at half capacity. Freight cargo companies increase their profits by optimizing their operations; hence, companies are looking to load their trucks as much as possible. This policy is detrimental to the integrity of road infrastructure.

4.4. Models of basic load spectra by regions of Colombia

The load spectra, for different axle types, were modeled using a mixed distribution approach for axles transiting through roads located on the principal regions of Colombia. Calibrated statistical models are presented in Figures 7 and 8. The modeling was carried out for different regions of the Colombian territory. The analysis focused on the tandem and tridem axles considering that these present a wider separation between the peaks on the load spectra, hence would require the use of a third distribution.

Chi-square test was used to determine the goodness of fit of the proposed models. This method compares the difference in magnitudes between the observed and modeled data. Results presented on Figures 7 and 8 indicate that there is no statistical significant difference between the two sets of distributions that were evaluated, the observed distributions and the modeled distributions. P-values less than 0.05 would suggest the rejection of the null hypothesis, however, the values presented on Figures 7 and 8 are significantly higher than 0.05. These results suggest that the proposed models adequately represent the axle load spectra of the different regions of Colombia. Furthermore, the coefficient of determination ($R^2$ values) was obtained in a complementary form for each model. It can be seen that for all cases, $R^2$ values are close to 1, which suggest that the models present sufficient explanatory power to represent the behavior of the entire population.

4.5. Relationship between types of products transported and overloads

Figure 9 shows the frequency distribution by type of products transported under conditions of overload within
5. Conclusions

An evaluation of truck overloads on the Colombian highway network was conducted. Truck data associated with various vehicle categories were obtained from thirty-eight (38) mobile stations that were placed in different strategic points of the Colombian national road network to characterize the traffic condition of the country. An overview of loads transiting the Colombian territory was evaluated; results showed that a large percentage of axles with overloads circulate over Colombian roads. A detailed description of the shape and the practical significance of different regions of Colombia; Table 2 details the overall information of the evaluated trucks. These results suggest that the types of products that are commonly associated with overloads are related to mining, agriculture and industrial use.

Furthermore, it is clear that the Caribbean and the Andean region are experimenting significant levels of overloads, with 14% and 7% respectively. This situation should encourage local agencies to implement control measures that guarantee that vehicles circulate without overloads, especially in the Caribbean and the Andean region.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Description of axles obtained from the mobile weighing operatives</th>
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<tbody>
<tr>
<td><strong>Truck Information</strong></td>
<td><strong>Region</strong></td>
</tr>
<tr>
<td></td>
<td>Andean</td>
</tr>
<tr>
<td>Number of trucks evaluated</td>
<td>45916</td>
</tr>
<tr>
<td>Number of trucks with overloads</td>
<td>3057</td>
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<tr>
<td>Overload %</td>
<td>7%</td>
</tr>
</tbody>
</table>

**Figure 7** Load spectra and models of load spectrum for the Andean and Caribbean regions of Colombia
the load spectra obtained for the single directional axles, single standard axles, tandem and tridem axles of the Colombian Andean region was presented. In addition, load spectra were modeled using a mixed distribution approach for axles circulating roads located on the principal regions of Colombia.

A mixed distribution model consisting of three statistical distributions was used to simulate the full form of the load spectra; two Log-normal probability functions to capture the peaks of the spectrum and a Normal probability function to capture the intermediate distribution between the two peaks. The analysis focused on the tandem and tridem axles considering that these present a wider separation between the peaks on the load spectra, hence require the use of a third distribution. Results showed that the models present sufficient explanatory power to represent successfully the load spectra of the evaluated roads.

Moreover, an analysis was performed to evaluate the types of goods transported in overload conditions; frequency distributions were developed by product type in order to identify economic sectors that contribute to the deterioration of roads throughout the country. It was found that the industrial, agricultural and mining products are the most common goods associated with overloads; particularly, the transportation of coal contributes to the significant deterioration observed in pavement structures. Finally, it was found that a large proportion of vehicles travel with overloads; a significant proportion of the vehicles do not comply with the provisions defined on the Resolution No. 4100 of December 28, 2004, issued by the Ministry of Transport of Colombia. One possible cause of this phenomenon is derived from the difference between the income earned by transporting goods in overload conditions and the value of the penalty, which is significantly lower. Under this condition, transporters prefer to travel with overloads even if it means that they have to pay a fine, considering that the punitive policy in Colombia associated with overload does not lead to the immobilization of the offending vehicle.

It is recommended that agencies responsible for the condition of the Colombian road infrastructure define a clear policy for overload control in the country; the implementation of tougher sanctions on vehicles that do not comply with the regulations, since these vehicles undermine the integrity of the country’s infrastructure. In addition, it should be ensured that the funds collected from these sanctions are destined at repairing pavement structures.

![Figure 8 Load spectra and models of load spectrum for the Pacific and Orinoquia regions of Colombia.](image)

<table>
<thead>
<tr>
<th></th>
<th>Pacific Region</th>
<th>Orinoquia Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tandem Axle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(μ₁)</td>
<td>2.422</td>
<td>1.905</td>
</tr>
<tr>
<td>Ln(μ₂)</td>
<td>3.063</td>
<td>3.028</td>
</tr>
<tr>
<td>μ₃</td>
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<td>18</td>
</tr>
<tr>
<td>σ₁</td>
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<td>p₁</td>
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<td>p₂</td>
<td>0.543</td>
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<tr>
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<tr>
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<tr>
<td>x²_a</td>
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<td>R²</td>
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<td>P-value</td>
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</tr>
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</table>

| Tridem Axle |                |                 |
| Ln(μ₁)     | 3.20           | 1.823           |
| Ln(μ₂)     | 3.00           | 3.195           |
| μ₃         | -              | 22.16           |
| σ₁         | 0.072          | 0.139           |
| σ₂         | 0.17           | 0.033           |
| σ₃         | -              | 2.771           |
| p₁         | 0.63           | 0.325           |
| p₂         | 0.37           | 0.362           |
| χ²_OBS     | 0.024          | 3.00            |
| α          | 0.05           | 0.05            |
| x²_a       | 14.067         | 14.067          |
| R²         | 0.98           | 0.968           |
| P-value    | 0.999          | 0.884           |
6. Acknowledgements

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7. References


