INTRODUCTION

Foams are useful in many industrial applications, such as in paper processing, fire-fighting, and enhanced oil recovery (EOR). However, in other processes the production of foam is often an unwanted consequence.

A proper design of a foam flow system requires an understanding of the physical phenomena involved. The nature of the foam, i.e., its structure, foamability, stability, and its overall rheological properties can determine both the economical and technical successes of an industrial process.

Several models have been developed for foam characterization to relate the different effects observed on foams. Scientists who have studied foamy fluids rheology have used different models for foam characterization. Utilizing the model that better represents the foam flow behavior enables more accurate prediction of the pressure drop. This requires that all the characteristic parameters of the foam are known.
Rheology can be evaluated using different equipments. Among the most commonly used are: Couette rheometer, circulating pipe rheometer and single-pass pipe rheometer. In the present study, a single-pass pipe rheometer was used for foam characterization due to its simple design and suitability for the flow conditions that were covered in this research.

Foam flow in straight pipes has been the subject of several investigations, such as: Calvert and Nezhati (1986); Calvert (1990); Enzendorfer et al. (1995); Hanselmann and Windhab (1996), and Deshpande and Barigou (2000). However, no specific correlation between the flow conditions and the rheology of the foam has yet been established. This is the gap the present study attempts to fill.

**EXPERIMENTAL WORK**

This study was carried out at the Tulsa University Separation Technology Projects (TUSTP). The rheology section, installed in a two-phase flow facility, consisted of 56 ft long 1 in. and 2 in.-diameter horizontal pipes (Figure 1). Two absolute pressure transducers are placed one at each end of the rheology section, for measurement of the pressure at the inlet and at the outlet of the pipes, enabling determination of the foam flow pressure drop. Also, a visualization window is installed in each pipe for flow pattern observations.

Water and surfactant are mixed in a tee to form the surfactant solution. This solution and compressed air are combined in a mixing-tee to promote foam formation. The generated foam flows through a static mixer to ensure homogeneity of the flow. The foam then flows through the rheology section where all measurements are acquired. Downstream, the flow enters into a separation tank where the air is vented into the atmosphere and the surfactant solution is drained.

Data were acquired for various inlet flow conditions: superficial gas velocity between 20 and 70 ft/s and superficial liquid velocity between 0.25 and 0.85 ft/s, using commercial surfactant (F-450) at two concentrations of 0.02 and 0.04 vol./vol..

**Foam Flow Pattern Maps**

Experimental flow pattern maps for aqueous foam flow in the 2 in.-diameter horizontal pipe of the rheology section were obtained. Three different flow patterns were observed: stratified, slug and annular flow. Figures 2 and 3 show the flow pattern maps for the two surfactant concentrations, namely, $C_s = 0.02\%$ and $C_s = 0.04\%$ vol./vol., respectively.

As can be seen, the results for the two surfactant concentrations are similar. Also, the Taitel and Dukler (1976) model (represented by the solid lines) does not predict accurately the stratified to non-stratified and the slug to annular transition boundaries for foam flow. Modifications of the original model are required for matching the experimental data.

Note that flow patterns were also observed in the 1 in.-diameter rheology pipe section. For this case, only annular flow was observed for all flow conditions.

**Foam Rheology in Pipes**

The experimental data for the 2 in.-diameter pipe for the $C_s = 0.02\%$ vol./vol. case are presented in figure 4 in the form of the measured pressure drop ($\Delta p$) on the y-axis vs. the superficial gas velocity ($v_{SG}$) on the x-axis, whereby the parameter is the superficial liquid velocity ($v_{SL}$). An error analysis for the pressure drop measurement was carried out and the corresponding uncertainty is included with the reported results in Figure 4. As expected, the pressure drop increases when either the gas or the liquid flow rates are increased. For the analysis of the rheological behavior of the foam flow, the experimental data are re-plotted in Figures 5 and 6. Figure 5 presents the experimental data in the form

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*Figure 1. Schematic of Rheology Section.*
Figure 2. Foam Flow Pattern Results in 2 in. Diameter Rheology Pipe Section for $C_s = 0.02\%$ Plotted on Taitel and Dukler (1976) Map.

Figure 3. Foam Flow Pattern Results in 2 in. Diameter Rheology Pipe Section for $C_s = 0.04\%$ Plotted on Taitel and Dukler (1976) Map.

of the wall shear stress vs. shear rate. The shear stress is determined as $\tau_w = \frac{d}{4} \frac{dp}{dL} = \frac{d}{4} \Delta p / L$, where $\Delta p$ is the pressure drop across the rheology section pipe and $L$ is the distance between the two pressure transducers. As can be seen from the figure, foam flow does not behave as a Newtonian fluid.

The experimental data are plotted in figure 6 in the form of natural log of wall shear stress vs. natural log of shear rate. The figure demonstrates that there are two different regions; one corresponding to stratified flow conditions while the other is for non-stratified (slug and annular) flow conditions. Within each region, the flow behaves according to the Power Law model, as the data exhibit straight lines, with different slopes for the stratified and the non-stratified cases.

Similar results are obtained for $C_s = 0.04\%$ vol./vol., whereby the data for this case, too, demonstrates that foam flow behaves as a Power Law fluid, with different slopes for the stratified and non-stratified cases, and different slopes as compared with the lower concentration results given in Figure 6.
Figure 4. Foam Flow Pressure Drop vs. Gas Flow Rate Results
\((C_s = 0.02\%, d = 2\text{ in.})\).

Figure 5. Foam Flow Wall Shear Stress vs. Shear Rate Results
\((C_s = 0.02\%, d = 2\text{ in.})\).

Figure 6. Foam Flow Wall Shear Stress vs. Shear Rate Results in Logarithmic Scale \((C_s = 0.02\%, d = 2\text{ in.})\).
MODELING

Foam Rheology in Pipes

Various rheological models were tested against the experimental data of foam flow acquired in this study. The best correlation with the data was obtained with the Power Law Model, as shown in figure 6.

For the Power Law Model, the shear stress vs. shear rate is given by:

$$\tau = K_f \left( \frac{8 \cdot \nu_M}{d} \right)^n \quad (1)$$

where $K_f$ is the consistency index and $n$ is the flow behavior index.

For this study, $K_f$ is determined from the experimental data as follows:

$$K_{f\text{EXP}} = \left( \frac{\tau_W}{\left( \frac{8 \cdot \nu_M}{d} \right)^n} \right) \quad (2)$$

where the wall shear stress can be determined from the experimental pressure gradient data, namely, $\tau_W = d / 4 \times dp / dL$.

The flow behavior index, $n$, can be obtained from the slope of the curve, namely, ln($\tau_W$) vs. ln($\delta\nu_M$/d), as presented in figure 6. As only a slight difference occurs between $n$ for stratified and non-stratified flow, an average $n$ value is obtained for each surfactant concentration: $n = 1.18$ for $C_s = 0.02\%$, and $n = 1.08$ for $C_s = 0.04\%$.

The following equations were obtained by correlating the experimental data, using the form of the Law of the Wall. The consistency index for the stratified flow regime is given by:

$$K_{fS} = a_3 \ln(Re_M) + b_3, \quad (3)$$

which is valid for the range:

$$4 \times 10^3 < Re_{SL} < 2 \times 10^4 \text{ and } 10^3 < Re_M < 10^6.$$  

Similarly, for non-stratified flow:

$$K_{fNS} = a_{NS} \ln(Re_M) + b_{NS} + Re_M^{f}, \quad (4)$$

valid for the range $8 \times 10^3 < Re_{SL} < 3 \times 10^4$ and $10^3 < Re_M < 10^7$.

The coefficients $a_i$ and $b_i$ for both the stratified and non-stratified flow conditions can be determined from Eqs. 5 and 6, and the exponent $f = -3.508$ is a constant, for all cases:

$$a_i = A_i \cdot \ln(Re_{SL}) + A_2, \quad (5)$$
$$b_i = B_i \cdot \ln(Re_{SL}) + B_2, \quad (6)$$

The coefficients $A_i$ and $B_i$ are flow pattern dependent and are given in Table 1 for both $C_s = 0.02\%$ and $C_s = 0.04\%$.

The superficial liquid Reynolds number and the mixture Reynolds number are defined, respectively, by

$$Re_{SL} = \frac{\rho_L \cdot \nu_{SL} \cdot d}{\mu_L}, \quad (7)$$

and

$$Re_M = \frac{\rho_L \cdot \nu_M \cdot d}{\mu_L}. \quad (8)$$

The definition of the mixture Reynolds number is similar to the one presented by Briceño and Joseph (2003) for self-lubrication transport of aqueous foams.

| Table 1. Values of Rheology Correlation Coefficients. |
|---------------------------------|---------------------------------|
| $C_s = 0.02\%$ | $C_s = 0.04\%$ |
| Stratified | Non-Stratified | Stratified | Non-Stratified |
| $A_1$ | 0.242 | -0.510 | -0.824 | 1.282 |
| $A_2$ | -2.263 | 4.647 | 6.953 | -10.390 |
| $B_1$ | -2.289 | 7.676 | 12.475 | -14.698 |
| $B_2$ | 22.524 | -68.893 | -104.335 | 119.002 |
Flow Pattern Prediction

Stratified to non-Stratified Transition

Figures 2 and 3 show that Taitel and Dukler (1976) model underpredicts this transition boundary, namely, the predicted transition occurs at lower superficial velocities, as compared with the experimental data.

For the transition between stratified to non-stratified flow, Taitel and Dukler (1976) used a smooth interface approach, namely, \( f_i = f_G \). For stratified wavy flow, a constant interfacial friction factor was proposed, i.e., \( f_i = 0.0142 \) (Shoham and Taitel, 1984). In foam flow, a higher interfacial friction factor occurs due to the rough foamy interface. In this study, the following interfacial friction factor is proposed for foam flow conditions.

\[
f_i = 0.02 \tag{9}
\]

Slug to Annular Transition

As can be seen in figures 2 and 3, this transition boundary is also underpredicted by the original model. The original Taitel and Dukler (1976) model proposed that this transition occurs when the stratified flow configuration is unstable and the equilibrium liquid level is \( h_L / d = 0.5 \). This criterion was later modified by Bamea et al. (1980), accounting for an average liquid holdup in the slug, namely, 0.7, yielding \( h_L / d = 0.5 \times 0.7 = 0.35 \) as the criterion for this transition. However, for foam flow lower values of liquid holdup may occur in the slug. In this study, a value of 0.26 is assumed for the liquid holdup in the slug (corresponding to rhombohedra packing of gas bubbles), resulting in the following criterion for the transition from slug to annular under foam flow conditions.

\[
\frac{h_L}{d} = 0.5 \times 0.26 = 0.13 \tag{10}
\]

RESULTS

Foam Rheology in Pipes

Figure 7 shows a comparison between the experimental data and the predictions of the extension of the Power Law Model for foam flow for the \( C_s = 0.02\% \) surfactant concentration case. The y-axis is the pressure drop across the test section, \( \Delta p \) in psi. The x-axis and the parameter are chosen based on the developed correlation as \( \text{Re}_d \) and \( \text{Re}_{SL} \), respectively. The solid lines represent the predictions of the proposed correlation for both stratified and non-stratified flow conditions. As can be seen, good agreement is observed between the experimental data and correlation predictions.

To demonstrate the accuracy of the proposed foam rheology correlation, a comparison between the measured pressure drop, \( \Delta p_{\text{exp}} \), and the predicted pressure drop, \( \Delta p_{\text{cor}} \), was carried out. The measured and predicted pressure drop values agree within \( \pm 10\% \) for both surfactant concentration cases.

Flow Pattern Prediction

Figure 8 presents a comparison between the predictions of the original Taitel and Dukler (1976) model, the modified model based on Eqs. 9 and 10, and the experimental data. As can be observed, the predictions of the modified model agree closely with the experimental data.
CONCLUSIONS

The power law model was modified for aqueous foam flow. A correlation was developed for the prediction of the consistency index, $K_f$, which is flow pattern dependent, namely for stratified flow and non-stratified flow conditions.

The Taitel and Dukler (1976) model was modified for flow pattern prediction in foam flow. The modifications of the model include an interfacial friction factor of $f_I = 0.02$ for prediction of the equilibrium stratified flow and $h_L/d = 0.13$ as the criterion of transition boundary between slug and annular flow.

Comparison between the developed correlation and model modifications with the acquired experimental data shows good agreement.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support of Universidad Central de Venezuela and Tulsa University Separation Technology Projects (TUSTP).

NOMENCLATURE

- $A, a =$ coefficient (-)
- $B, b =$ coefficient (-)
- $C_s =$ surfactant concentration (%)
- $d =$ diameter (in)
- $f =$ rheology exponent (-)
- $f_I =$ interfacial friction factor (-)
- $h_L =$ height (in)
- $K_f =$ consistency index (psia.s$^n$)
- $n =$ flow behavior index
- $p_{AV} =$ average pressure (psia)
- $Re =$ Reynolds number
- $T =$ temperature ($^\circ$F)
- $v =$ velocity (ft/s)

Greek symbols:

- $\Delta p =$ pressure gradient (psi)
- $\mu_L =$ liquid viscosity (lbm/ft x s)
- $\theta =$ inclination angle ($^\circ$)
- $\rho_L =$ liquid density (lb/ft$^3$)
- $\tau_W =$ wall shear stress (psi)
- $\sigma =$ surface tension (dynes/cm)

Subscripts:

- EXP = experimental
- M = mixture
- NS = non-stratified
- S = stratified
- SG = superficial gas
- SL = superficial liquid

Abbreviations:

- A = annular flow
- mod = model
- dat = experimental data
- SF = stratified flow
- SL = slug flow

REFERENCES


