Wholesale price contract under fairness preference with random capacity and random demand

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Abstract

This paper investigates the effect of fairness preference on a dyadic supply chain with random capacity and random demand. Based on disadvantageous inequity aversion and advantageous inequity aversion, we establish the models of wholesale price contract with random capacity and stochastic demand. The retailer’s optimal ordering quantity is obtained. In addition, the impact of inequity aversion on the optimal order quantity and the supply chain coordination are also analyzed. We finally present numerical examples to illustrate the theoretical results. The results show that the wholesale price contract can’t achieve supply chain coordination for the risk-neutral retailer. However, when the retailer has extreme advantageous inequity aversion, the wholesale price contract can improve the profit of the whole supply chain and coordinate the supply chain.

Keywords: supply chain, random capacity, fairness preference, coordination.

1. INTRODUCTION

This paper probes into a dyadic supply chain with random capacity and random demand in which a supplier deals with a retailer. The supplier is risk-neutral and the retailer is assumed to have the fairness preference, which means that the retailer have the desire for a higher relative profit compared with the other party’s (Loch and Wu, 2008). The existing studies of behavioral operations management show that people pay much attention on fairness preference in their daily life (Fehr and Schmidt, 1999; Su, 2008; Liu and Zha, 2009). People not only are concerned about their self-interests, but also care about other people’s earnings (Rabin, 1993). With fairness preference, one may punish his (her) collaborator at the cost of decreasing his (her) own profits when unfairness is perceived. The behavior of fairness preference is incompatible with traditional utility theory since it is against the basic assumption that people are rational, while the existence of fairness preference is supported by many empirical studies and experiments (Du et al., 2014). Kahneman et al.(1986) point out that organizations have the fairness preference just like individuals in many cases. Therefore, just relying on the self-interest model cannot well explain the behavior of fairness preference, while introducing theoretical model of the fairness preference will have better persuasive in many cases.

It is still in its early stage to study on fairness preference. Inadequate effort so far has been paid to incorporate fairness preference into supply chain decision. Cui et al. (2007) analyze fairness preference in a conventional dyadic channel to investigate how fairness may affect channel coordination. They show that the supplier can use a simple constant
wholesale price above his marginal cost to coordinate this channel in the form of maximizing both profit and utility. However, their analyses are based on deterministic demand. Caliskan-Demirag et al. (2010) extend the results of Cui et al. to other nonlinear demand functions that are commonly used in the literature. Their results reveal that, compared to the linear demand, the exponential demand function requires less stringent conditions to achieve coordination when only the retailer is fairness-concerned. Ho and Zhang (2008) confirm the existence of fairness preference in the setting of supply chain. They put forward descriptive utility function rather than analyze the effects of fairness on coordination in detail. Xing et al. (2011) study the impact of fairness on strategies in dual-channel supply chain. They find that when retailers’ market share was relatively small, manufacturers would not care about whether retail channel was fair. However, when retailers’ market share was relatively large, manufacturer would pay attention to channel fairness preference in order to avoid the punishment that retailers set high retail price. Du et al. (2010) consider a dyadic supply chain with a newsvendor who has fairness preference. They analyze how the retailer’s preference of fairness may affect channel coordination in wholesale price contract, buy-back contract and revenue sharing contract respectively. Wang et al. (2014) make a comparative study of marketing channel multiagent Stackelberg model based on perfect rationality and fairness preference. They find different results if the retailers have a jealous fairness preference or have a sympathetic fairness preference. Bi et al. (2013) introduced inequity aversion to traditional two-stage supply chain. In their paper, however, the random capacity is not considered and the fairness reference point is still the supplier’s profit, which may deviate from reality.

In a complicated and changeable environment, supply chain management faces the risks of all kinds of uncertainty. The major source of randomness in supply chain models is the demand. However, the uncertainty of demand is not the only source of randomness. For example, Chinese Foxconn is a supplier of American Apple, because the workers don’t adapt to the stringent quality control standards of iphone 5, Zhengzhou’s factory happened strike in October 2012 (Chen and Chen, 2014). The incident affected the production and made the delivery quantity less than the order quantity for the month. In fact, in recent years, there has been a lot of emphasis on models with uncertainty of supply as well. These include yield loss (related to quality considerations), unreliable machinery and unplanned maintenance. We refer interest readers to Grosfeld-Nir and Gerchak (2004) and Yano and Lee (1995) for summaries of the effects of random supply on inventory control. Most of the literatures on supply randomness consider two models based on randomness in the capacity of the supplier and in the yield. An implicit assumption in almost all of the papers is the independence of demand and supply (Okyay et al., 2014). The main research of this paper is supply chain’s random capacity. Let $y$ be the amount ordered and $Q(y)$ be the amount received.

- Random Capacity: The supplier has some random replenishment capacity $K$ so that

$$Q(y) = \min\{K, y\}.$$ 

When an order is placed for $y$ units, the suppliers will ship $y$ if the total amount $K$ of on hand inventory that they poses is greater than $y$. Or else, they will send all the inventory they poses, which is $K$. Erdem and Özekici (2002) consider a periodically reviewed single-item inventory model in a random environment with random capacity and show that a base-stock policy is optimal.

Although the supply chain coordination is not a new problem, it is rare to simultaneously consider stochastic supply and random demand in the existing literatures. Gurnani and Gerchak (2007) study coordination of a decentralized assembly system in which the demand of the assembler is deterministic and the component yields are random. They present incentive alignment control mechanisms under which system coordination is
achieved. Yan et al. (2010) extend Gurnani and Gerchak’s model to the case of positive salvage value and \( n \) asymmetric suppliers, and show that the shortage penalty contract which can coordinate Gurnani and Gerchak’s model no longer coordinates the extended model. They present a new kind of contract, surplus subsidy contract, to coordinate the extended model and prove that the profit of the supply chain under coordination can be arbitrarily divided between the component suppliers and the assembler. Hu et al. (2013) investigate the coordination issue in a single-retailer and two-supplier supply chain under random demand and random supply with disruption. They find that the buy-back contract with a side payment to the backup supplier is provided to coordinate the supply chain. Wu et al. (2013) study the effect of capacity uncertainty on the inventory decisions of a risk-averse newsvendor with two well-known risk criteria, namely VaR included as a constraint and CVaR. They find that capacity uncertainty decreases the order quantity under the CVaR criterion. While under the VaR constraint, capacity uncertainty leads to an order decrease for low confidence levels, but to an order increase for high confidence levels. A recent paper by Inderfurth and Clemens (2014) analyze a buyer-supplier supply chain in which the buyer orders products from the supplier who suffers from random supply to meet a deterministic demand. They show that the double marginalization effect also exists under a simple wholesale price contract, whereas the supply chain coordination can be achieved with an overproduction risk sharing contract. All above literatures assume that all of the supply chain’s participants are risk-neutral or risk-averse, without considering the problem of fairness preference.

To the best of our knowledge, there has not been any published paper that has addressed such issue so far. Hence, our study sheds new light on the management of the fairness-concerned supply chain with random capacity and stochastic demand. Motivated by the observations above, we investigate a wholesale price contract in which random capacity and stochastic demand and fairness preference are jointly considered. The combined randomness of demand and supply enhances the level of uncertainty, thus leading to an increased complexity of the model. Our major contributions to the literature are as follows.

(1) The studies about fairness preference are still scarce despite the fact that such a model is much closer to reality than traditional studies based on rationality assumption. We investigate fairness preference in the supply chain with both demand and supply uncertainty from a new perspective. More attention is paid to individuals’ psychological perceptions.

(2) We improve the fairness reference point of Fehr and Schmidt’s model of inequity aversion, which is much closer to reality than before and the results of Bi et al. (2013) are special cases in our paper.

(3) We establish a new wholesale price contract model under fairness preference with random capacity and random demand. Our paper casts light on the impact of inequity-averse on decision and performance of the supply chain in the context of random capacity and stochastic demand. Meanwhile, we find the simple wholesale price contract can coordinate the supply chain as the retailer has extreme advantageous inequity aversion, which is impossible in risk-neutral assumption.

The rest of this paper is organized as follows. We formulate the basic model in non-fairness preference with random supply and stochastic demand in Section 2. In Section 3, we investigate the inequity-averse retailer’s optimal order policy based on disadvantageous inequity aversion and advantageous inequity aversion. The impact of inequity aversion on the retailer’s decision and the problem of wholesale price contract’s coordination are also examined. In Section 4, numerical experiments are provided. Finally, conclusions and suggestions for future research are given in Section 5.
2. Basic model in non-fairness preference

We study a dyadic supply chain with a single supplier and a single retailer in a single-period, random capacity and stochastic demand setting, as widely concerned in channel-related researches such as Bi et al. (2013), Okyay et al. (2014), etc.. Random demand and random capacity are assumed to be independent (Wu et al., 2013). Notations concerned in this paper are listed as follows:

- \(C_s\) and \(C_r\): the supplier’s production cost per unit and the retailer’s marginal cost per unit, let \(C=C_s+C_r\)
- \(\alpha\) and \(\beta\): the retailer and supplier’s penalty cost per unit, let \(\gamma=\alpha+\beta\)
- \(p\): selling price per unit, \(c<p\)
- \(v\): the retailer’s salvage value per unit, \(v<c\)
- \(w\): the supplier’s wholesale price
- \(q\): the retailer’s order quantity
- \(Q(q)\): amount of the retailer received
- \(D\): random demand. Its probability density function is \(f(x)\) and cumulative distribution function is \(F(x)\)
- \(K\): random capacity, i.e., the supplier has some random replenishment capacity so that \(Q(q) = \min\{K, q\}\). Its probability density function is \(g(y)\) and cumulative distribution function is \(G(y)\)
- \(q^*\): the retailer’s optimal order quantity
- \(q^0\): the supply chain’s optimal order quantity
- \(x^+=\max\{x, 0\}\): the positive part of \(x\)

Without loss of generality, we assume that \(F\) and \(G\) are differentiable, strictly increasing and \(F(0) = G(0) = 0\). Let \(F_0(\cdot) = 1 - F(\cdot)\) and \(G_0(\cdot) = 1 - G(\cdot)\).

Let \(S(K,q)\) be expected sales, \(\min\{D, K, q\}\),

\[
S(K,q) = E\left(\min\{D, K, q\}\right) = \int_{0}^{\infty} \int_{0}^{\min\{y,q\}} xdF(x) + \int_{\min\{y,q\}}^{\infty} ydF(x) dG(y) \tag{1}
\]

\[
= \begin{cases} 
\int_{0}^{q} xdF(x) + q \int_{q}^{\infty} dF(x) & y > q, \\
\int_{0}^{\infty} xdF(x) + y \int_{0}^{\infty} dF(x) & y \leq q.
\end{cases}
\]

Let \(I(K,q)\) be expected left over inventory, \((\min\{K,q\} - D)^+\),

\[
I(K,q) = E\left(\min\{K, q\} - D\right)^+ = E\left(\min\{K, q\}\right) - S(K,q). \tag{2}
\]

Let \(L(K,q)\) be the lost sales function, \((D - \min\{K,q\})^+\),

\[
L(K,q) = E\left(D - \min\{K, q\}\right)^+ = \mu - S(K,q). \tag{3}
\]

With a wholesale price contract the supplier charges the retailer \(w\) per purchased, thus the expected transfer payment from the retailer to the supplier is \(T(K,q) = wE(\min\{K,q\})\).

We refer interest readers to Lariviere and Porteus (2001), Cachon (2003), etc. for a more complete analysis of this contract in the context of newsvendor problem.

The retailer’s profit function is
\[ \pi_s(q) = pS(K, q) + vI(K, q) - c_sE(\min\{K, q\}) - \alpha L(K, q) - wE(\min\{K, q\}) \\
= (p - v + \alpha)S(K, q) - (c_s - v + w)E(\min\{K, q\}) - \alpha \mu. \]  

(4)

The supplier’s profit function is

\[ \pi_s(q) = wE(\min\{K, q\}) - c_sE(\min\{K, q\}) - \beta L(K, q) \\
= (w - c_s)E(\min\{K, q\}) - \beta \mu + \beta S(K, q). \]  

(5)

The supply chain’s profit function is

\[ \pi(q) = (p - v + \gamma)S(K, q) - (c - v)E(\min\{K, q\}) - \gamma \mu. \]  

(6)

Note that for any random variable \( K \) with a probability density function \( g \), we can write

\[ E(\min\{K, q\}) = \int_q^\infty yg(y)dy + q\int_q^\infty g(y)dy \]

and one can easily show that

\[ \frac{\partial E(\min\{K, q\})}{\partial q} = \tilde{G}(q). \]  

(7)

We take the derivative of \( S(K, q) \) with respect to \( q \), then we have

\[ \frac{\partial S(K, q)}{\partial q} \left\{ \begin{array}{ll} \int_q^\infty dF(x) & y > q \\ 0 & y \leq q \end{array} \right\} = \int_q^\infty dF(x)\int_y^\infty dG(y) = F(q)\tilde{G}(q). \]  

(8)

Using (7) and (8), we take the derivative of (6) with respect to \( q \) and set it equal to zero. Hence, we obtain the following first order condition

\[ \frac{\partial \pi(q)}{\partial q} = (p - v + \gamma)\tilde{F}(q)\tilde{G}(q) - (c - v)\tilde{G}(q) = 0. \]

Therefore, the optimal order quantity with random capacity and random demand of whole supply chain is obtained as

\[ q^* = F^{-1}\left( \frac{p - c + \gamma}{p - v + \gamma} \right). \]  

(9)

Similarly, using (7) and (8), we take the derivative of (4) with respect to \( q \) and set it equal to zero. Hence, we obtain the first order condition as follow

\[ \frac{\partial \pi_s(q)}{\partial q} = (p - v + \alpha)\tilde{F}(q)\tilde{G}(q) - (c_s - v + w)\tilde{G}(q) = 0. \]  

(10)

Therefore, the retailer’s optimal order quantity with random capacity and random demand is obtained as
Under the risk-neutral criterion, it is well known that the optimal order quantity of the standard newsvendor model with unlimited capacity is also (11), i.e., the uncertainty of capacity does not affect the newsvendor’s optimal order quantity. Indeed, this result is intuitive, as the newsvendor receives a lower profit by ordering less, and gains nothing by ordering more if the capacity is uncertain.

**Proposition 1.** The wholesale price contract can’t coordinate the risk-neutral supply chain with random capacity and random demand.

**Proof.** If and only if \( q^* = q^0 \), the supply chain coordination can be achieved. From (9) and (11), we easily get

\[
    w = \frac{p - v + \alpha}{p + \gamma - v} (c - v) - (c_r - v) < c_r, \quad (12)
\]

which implies that the supplier’s profit is non-positive. Therefore, it is not possible to coordinate the supply chain at this moment.

### 3. Behavioral model based on fairness preference theory

There are many models that describe people’s fairness preference psychology, such as FS model (Fehr and Schmidt, 1999), Rabin model (Rabin, 1993), DK model (Dufwenberg and Kirchsteiger, 2004) and the ERC model (Bolton and Ockenfels, 2000) and so on. This paper will be based on FS model and we will make further improvement to it. We will use a relative fairness reference point to describe the behavior of retailer’s fairness preference, which may be more realistic.

#### 3.1. The Model of Inequity Aversion

Fehr and Schmidt (1999) presented inequity aversion model in 1999 to describe the fairness preference of all participants. The model assumes the following. In addition to purely selfish players, there are players who dislike inequitable outcomes. They experience inequality if they are worse off in material terms than the other players in the experiment, and they also feel inequity if they are better off. Formally, consider a set of \( n \) players indexed by \( i \in \{1,2,\ldots,n\} \), and let \( x=(x_1,x_2,\ldots,x_n) \) denote the vector of monetary payoffs. The utility function of player \( i \in \{1,2,\ldots,n\} \) is given by

\[
    u_i(x) = x_i - a_i \frac{1}{n-1} \sum_{j\neq i} \max(x_j-x_i,0) - b_i \frac{1}{n-1} \sum_{j\neq i} \max(x_i-x_j,0). \quad (13)
\]

The second term measures the utility loss from disadvantageous inequity, while the third term measures the loss from advantageous inequity. Loewenstein et al. (1989) verify that players suffer more from inequity that is to their material disadvantage than from inequity that is to their material advantage. Therefore, we assume that \( a_i \geq b_i \) and \( 0 \leq b_i \leq 1 \). In the two-player case (13) simplifies to

\[
    u_i(x) = x_i - a_i \max(x_j-x_i,0) - b_i \max(x_i-x_j,0), \quad i \neq j. \quad (14)
\]

In this paper, the inequity aversion model is applied to the above dyadic supply chain. We assume that the retailer is inequity-averse and the supplier is risk-neutral. From (14) we can see that the retailer’s fairness reference point is the supplier’s profit.
However, this kind of fairness reference point has its limitations in reality because it requires absolute fairness. Taking into account that both the position and the contributed profit for entire supply chain is different. The retailer should use a certain proportion of the supply chain profit as fair reference point. Suppose deserving profit of the retailer is \( \theta_\pi(q) \), \( 0 \leq \theta \leq 1 \). We assume \( \theta_\pi(q) \) as fairness reference point for retailer is more appropriate to formulate the individuals’ fairness perceptions, because it focuses on the relative fairness by self-enforcingly integrating players’ power and contribution instead of the absolute fairness merely via several exogenous parameters as individuals’ inherent or default properties. In this case, the retailer’s utility function can be written in the form of the following piecewise functions

\[
u_r(\pi)=\begin{cases} 
\pi_r-a(\theta_\pi-\pi_r) & \theta_\pi \geq \pi_r, \\
\pi_r+b(\theta_\pi-\pi_r) & \theta_\pi < \pi_r.
\end{cases}
\]  

\( (15) \)

### 3.2. The model of disadvantageous inequity aversion

When \( \theta \geq \pi \), the retailer is disadvantageous inequity aversion and the retailer’s utility function is

\[
u_r(\pi)=(1+a)\pi_r-a\theta_\pi, a \geq 0.
\]

Putting (4) and (6) into the above equation, it comes to

\[
u_r(\pi)=\left[(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)\right]S(K,q) \\
+\left[a\theta(c-v)-(1+a)(c,-v+w)\right]E\left(\min[K,q]\right) + \left[a\theta\mu-(1+a)\alpha\mu\right].
\]  

\( (16) \)

**Proposition 2.** When the coefficient of disadvantageous inequity aversion \( a \) and \( \theta \) satisfy \( \frac{a}{1+a} < \frac{p-v+\alpha}{\theta(p-v+\gamma)} \), and \( \theta \geq \pi \), there exists a unique optimal order quantity \( q^* \) that maximizes the expected utility of disadvantageous inequity-averse retailer with random capacity and random demand and satisfies the following equation

\[
q^*=F^{-1}\left(\frac{(1+a)(p+\alpha-c,-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}\right).
\]  

\( (17) \)

**Proof.** Taking the partial derivative of \( u_r(\pi) \) with respect to \( q \), we get the following first-order condition

\[
\frac{\partial u_r(\pi)}{\partial q} = \left[(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)\right]F(q)\bar{G}(q) \\
+\left[a\theta(c-v)-(1+a)(c,-v+w)\right]\bar{G}(q) = 0.
\]

Then we can derive

\[
q^*=F^{-1}\left(\frac{(1+a)(p+\alpha-c,-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}\right).
\]  

Let
When the coefficient of disadvantageous inequity aversion $\alpha$ satisfies
\[ \frac{a}{1+a} < \frac{p-v+\alpha}{\theta(p-v+\gamma)} \text{, we have } \frac{\partial \tau(q)}{\partial q} < 0. \]

Note that $\bar{\tau}(q) > 0$, hence,
\[ \tau(q) \overline{G}(q). \]

When $q > F^{-1}\left(\frac{(1+a)(p+\alpha-c_r-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}\right)$, we have
\[ \frac{\partial u_r(\pi)}{\partial q} < 0. \]

When $q < F^{-1}\left(\frac{(1+a)(p+\alpha-c_r-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}\right)$, we have
\[ \frac{\partial u_r(\pi)}{\partial q} > 0. \]

Therefore, $u_r(\pi)$ is concave in $q$ and $q^*$ is the retailer’s unique optimal order quantity.

**Proposition 3.** When the coefficient of disadvantageous inequity aversion $\alpha$ and $\theta$ satisfy
\[ \frac{a}{1+a} < \frac{p-v+\alpha}{\theta(p-v+\gamma)} \text{ and } \theta \geq \pi, \]
the disadvantageous inequity-averse retailer’s optimal order quantity is no greater than its risk-neutral counterpart, and the retailer’s optimal order quantity $q^*$ is decreasing in $\alpha$.

**Proof.** Taking the first-order derivative of $q^*$ with respect to $a$ and applying the chain rule, we get
\[ \frac{\partial q^*}{\partial a} = \frac{dF^{-1}}{dx} \frac{\partial x}{\partial a}, \]
where
\[ x = \frac{(1+a)(p+\alpha-c_r-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}. \]

Note that $F(x)$ is strictly increasing and $F(x)$ and $F^{-1}(x)$ have the same monotonic, we have $\frac{dF^{-1}}{dx} > 0$, while
\[ \frac{\partial x}{\partial a} = -\frac{\theta(p-v+\alpha)(w-c_r)+\theta\beta(c_r-v+w)}{[(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)]^2} < 0. \]

Hence, $\frac{\partial q^*}{\partial a} = \frac{dF^{-1}}{dx} \frac{\partial x}{\partial a} < 0$, which implies that the retailer’s optimal order quantity $q^*$ is decreasing in $\alpha$.

Since $q^*$ and $a$ are negatively correlated, thus when $\alpha = 0$, we have
\[ q^* = F^{-1}\left(\frac{(1+a)(p+\alpha-c_r-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)}\right) \leq F^{-1}\left(\frac{p+\alpha-c_r-w}{p-v+\alpha}\right). \]

Therefore, by (11), we get the disadvantageous inequity-averse retailer’s optimal order quantity is no greater than its risk-neutral counterpart.
Proposition 3 states that the disadvantageous inequity-averse retailer’s optimal order quantity will be conservative compared with only concerned with personal interests’ retailer. The higher degree of inequity aversion the retailer is, the more conservative his decision to order. This is because when the profit below the fairness reference point, the retailer will produce a “jealous” mentality. The retailer will decrease the optimal order quantity to reduce inequality of income.

**Proposition 4.** Even when the coefficient of disadvantageous inequity aversion \( a \) and \( \theta \) satisfy \( \frac{a}{1+a} < \frac{p-v+\alpha}{\theta(p-v+\gamma)} \) and \( \theta \geq \pi \), the wholesale price contract still cannot coordinate the supply chain.

**Proof.** If and only if \( q^* = q^0 \), the supply chain coordination can be achieved. That is

\[
F^{-1}\left( \frac{(1+a)(p+\alpha-c,-w)-a\theta(p-c+\gamma)}{(1+a)(p-v+\alpha)-a\theta(p-v+\gamma)} \right) = F^{-1}\left( \frac{p-c+\gamma}{p-v+\gamma} \right).
\]

Note that \( a \geq 0 \), we have

\[
w = c_s - \frac{(c-v)}{p + \gamma - v} \beta < c_s,
\]

which implies that the supplier’s profit is non-positive. Therefore, it is not possible to coordinate the supply chain at the moment.

### 3.3. The model of advantageous inequity aversion

When \( \theta \pi < \pi \), the retailer is advantageous inequity aversion and the retailer’s utility function is

\[
u_r(\pi) = (1-b)\pi_r + b\theta \pi, \quad 0 \leq b \leq 1.
\]

Putting (4) and (6) into the above equation, it comes to

\[
u_r(\pi) = \left[ (1-b)(p-v+\alpha)+b\theta(p-v+\gamma) \right] S(K,q)
\] \[
- \left[ (1-b)(c_s-v+w)+b\theta(c-v) \right] E\left( \min\{K,q\} \right) - (1-b)\alpha \mu + b\theta \mu.
\]

**Proposition 5.** When \( \theta \) satisfies \( \theta \pi < \pi \), there exists a unique optimal order quantity \( q^* \) that maximizes the expected utility of advantageous inequity-averse retailer with random capacity and random demand and satisfies the following equation

\[
q^* = F^{-1}\left( \frac{(1-b)(p+\alpha-c,-w)+b\theta(p-c+\gamma)}{(1-b)(p-v+\alpha)+b\theta(p-v+\gamma)} \right).
\]

**Proof.** Taking the partial derivative of \( u_r(\pi) \) with respect to \( q \), we get the following first-order condition

\[
\frac{\partial u_r(\pi)}{\partial q} = \left[ (1-b)(p-v+\alpha)+b\theta(p-v+\gamma) \right] \tilde{F}(q) \tilde{G}(q)
\] \[
- \left[ (1-b)(c_s-v+w)+b\theta(c-v) \right] \tilde{G}(q) = 0.
\]
Then we can derive
\[ q^* = F^{-1}\left(\frac{(1-b)(p+c_w-c_v-w)+b\theta(p-c+v)}{(1-b)(p-v+c)+b\theta(p-v+\gamma)}\right). \]

Let
\[ \frac{\partial u_s(\pi)}{\partial q} = \left\{ [(1-b)(p+v+c)+b\theta(p+v+\gamma)]F(q) - [(1-b)(c_v-v+w)+b\theta(c-v)] \right\} \bar{G}(q) \]
\[ = \tau(q)\bar{G}(q). \]

We have \( \frac{\partial \tau(q)}{\partial q} = -\left[(1-b)(p+v+c)+b\theta(p+v+\gamma)\right]f(q) < 0. \) Note that \( \bar{G}(q) > 0, \) hence,

when \( q > F^{-1}\left(\frac{(1-b)(p+c_w-c_v-w)+b\theta(p-c+v)}{(1-b)(p-v+c)+b\theta(p-v+\gamma)}\right), \) \( \frac{\partial u_s(\pi)}{\partial q} < 0; \)

when \( q < F^{-1}\left(\frac{(1-b)(p+c_w-c_v-w)+b\theta(p-c+v)}{(1-b)(p-v+c)+b\theta(p-v+\gamma)}\right), \) \( \frac{\partial u_s(\pi)}{\partial q} > 0. \)

Therefore, \( u_s(\pi) \) is concave in \( q \) and there exists a unique optimal order quantity \( q^* \) satisfies (20).

**Proposition 6.** When \( \theta \) satisfies \( \theta < \pi, \) the advantageous inequity-averse retailer’s optimal order quantity is not less than its risk-neutral counterpart, and the retailer’s optimal order quantity \( q^* \) is increasing in \( b. \)

**Proof.** Taking the first-order derivative of \( q^* \) with respect to \( b \) and applying the chain rule, we get
\[ \frac{\partial q^*}{\partial b} = \frac{dF^{-1}}{dx} \frac{\partial x}{\partial b}, \]

where \( x = \frac{(1-b)(p+c_w-c_v-w)+b\theta(p-c+v)}{(1-b)(p-v+c)+b\theta(p-v+\gamma)} \).

Note that \( F(x) \) is strictly increasing and \( F(x) \) and \( F^{-1}(x) \) have the same monotonic, we have \( \frac{dF^{-1}}{dx} > 0, \) while
\[ \frac{\partial x}{\partial b} = \frac{\theta(p-v+c)+\theta\beta(c_v-v+w)}{[(1-b)(p-v+c)+b\theta(p-v+\gamma)]^2} > 0. \]

Hence, \( \frac{\partial q^*}{\partial b} = \frac{dF^{-1}}{dx} \frac{\partial x}{\partial b} > 0, \) which implies that the retailer’s optimal order quantity \( q^* \) is increasing in \( b. \)

Since \( q^* \) and \( b \) are positively correlated, thus when \( b = 0, \) we have
Therefore, by (11), we get the advantageous inequity-averse retailer’s optimal order quantity is not less than its risk-neutral counterpart.

Proposition 6 shows that the advantageous inequity-averse retailer’s optimal order quantity will order more than the risk-neutral retailer. The higher degree of inequity aversion the retailer is, the more his decision to order. This is because when the profit below the fairness reference point, the retailer will produce a “sympathetic” mentality. The retailer will increase the optimal order quantity to reduce inequality of income.

Proposition 7. When $\theta$ satisfies $\theta \geq \pi$ and the retailer has extreme advantageous inequity aversion, i.e., $b=1$, the simple wholesale price contract can achieve supply chain coordination.

Proof. If and only if $q^*=q^0$, the supply chain coordination can be achieved. That is

\[ F^{-1}\left(\frac{(1-b)(p+\alpha-c_w-w)+b\theta(p-c+\gamma)}{(1-b)(p-v+\alpha)+b\theta(p-v+\gamma)}\right) \geq F^{-1}\left(\frac{p+\alpha-c_w-w}{p-v+\alpha}\right). \]

When $b=1$, we have

\[ w = c_w - \frac{(c-v)}{p+\gamma} \beta < c_w, \]

which implies that the supplier’s profit is non-positive. Therefore, it is not possible to coordinate the supply chain at the moment.

When the retailer has extreme advantageous inequity aversion i.e., $b=1$,

\[ F^{-1}\left(\frac{(1-b)(p+\alpha-c_w-w)+b\theta(p-c+\gamma)}{(1-b)(p-v+\alpha)+b\theta(p-v+\gamma)}\right) = F^{-1}\left(\frac{p-c+\gamma}{p-v+\gamma}\right). \]

Therefore, for any $w \in [c_w, p-c_w]$, we have $q^*=q^0$, i.e., the simple wholesale price contract can coordinate supply chain.

The literature (Bi et al., 2013) point out that only when the coefficient of advantageous inequity aversion $b=0.5$, the wholesale price contract can coordinate the supply chain, while the condition of this paper is satisfying $b=1$. The reason is that the fairness reference point of the literature (Bi et al., 2013) is the supplier’s profit, which is continually changed as the distribution of whole supply chain’s profits, while the fairness reference point of this article is fixed and will not change as the distribution of the retailer and the supplier’s profits. Therefore, under the same profit distribution, the degree of the retailer’s inequity aversion of this paper is only half of the literature (Bi et al., 2013). This implies that the fairness preference criterion should be carefully selected as it has an important effect on inventory decisions.

4. Numerical experiments

In this section, we will perform the numerical experiments to illustrate our results.
4.1. The case of disadvantageous inequity aversion

Without loss of generality, the demand of market $D$ is assumed to follow a normal distribution, i.e., $D \sim N(500,100^2)$. Suppose that $K$ is uniformly distributed between 400 and 900. The other parameters are set as follows: $p=55; \alpha=\beta=10; \gamma=20; c_r=5, c_s=30, c=35; \nu=10; \theta=0.5; w=40$. Obviously, for any $\alpha \geq 0$, we have $\frac{a}{1+a} < \frac{p-v+\alpha}{\theta(p-v+\gamma)}$. From equations (9) and (6), we can calculate the optimal order quantity of the supply chain $q^0=529$ and the profit of the supply chain $\pi(q)=7360$. Let $a$ equal different values, we can solve the retailer’s optimal order quantities by Proposition 2, then substituting them into each profit functions and utility function. Results are presented in Table 1 and Figures 1-3.

As it can be seen from Table 1, the retailer is the case of disadvantageous inequity aversion at this moment (i.e., the retailer’s profits is less than half of the total supply chain profits). When the retailer is risk-neutral (i.e., $\alpha=0$), his optimal order quantity is 465, which is less than supply chain’s optimal order quantity $q^0=529$, the wholesale price contract can not coordinate the supply chain.

Figures 1 and 2 demonstrate that the retailer’s optimal order quantity and the supply chain’s profit are decreasing in the retailer’s degree of disadvantageous inequity aversion. The more disadvantageous inequity aversion the retailer is, the less he will order. The possible explanation is that the more the retailer orders, the larger gap profit he feels, which will enlarge the unfairness to him and thus reduce his utility. This situation obviously can not coordinate the supply chain.

From Table 1 and Fig. 3, we can find that when the retailer’s profit is lower than the fairness reference point, with the degree of disadvantageous inequity aversion increases, the retailer would rather sacrifice his own interest to punish supplier due to “jealous” mentality. Since the supplier’s profit decrease more quickly, the gap of retailer and supplier’s profit gradually reduce. However, this will lead to reduce the supply chain’s efficiency. So the decision-maker should be appropriate to consider the fairness preference to make the channel become more harmonious.

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<th>$\pi_s(q)$</th>
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<th>$u_r(\pi)$</th>
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Table 2 The influence of retailer’s advantageous inequity aversion on supply chain

<p>| | | | | |</p>
<table>
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</table>

Figure 1. The influence of $a$ on optimal order quantity $q^*$

Figure 2. The influence of $a$ on supply chain’s profit

Figure 3. The influence of $a$ on retailer and supplier’s profit

Figure 4. The influence of $b$ on optimal order quantity $q^*$

Figure 5. The influence of $b$ on supply chain’s profit

Figure 6. The influence of $b$ on retailer and supplier’s profit
4.2. The case of advantageous inequity aversion

This subsection’s parameters use the above supposes of subsection 4.1 except the selling price $p=70$. Similarly, from equations (9) and (6), we can calculate the optimal order quantity of the supply chain $q^0=549$ and the profit of the supply chain $\pi(q)=14333$. Let $b$ equal different values, we can solve the retailer’s optimal order quantities by Proposition 5, then substituting them into each profit functions and utility function. Results are presented in Table 2 and Figures 4-6.

As it can be seen from Table 2, the retailer is the case of advantageous inequity aversion at this moment (i.e., the retailer’s profits is more than half of the total supply chain profits). When the retailer is risk-neutral (i.e., $b=0$), his optimal order quantity is 500, which is less than supply chain’s optimal order quantity $q^0=549$, the wholesale price contract also can not coordinate the supply chain.

Figures 4 and 5 show that the retailer’s optimal order quantity and the supply chain’s profit are increasing in the retailer’s degree of advantageous inequity aversion. The more advantageous inequity aversion the retailer is, the more he will order. The possible explanation is that the less the retailer orders, the larger gap profit he feels, which will enlarge the unfairness to him and thus reduce his utility. In particular, when the retailer has extreme advantageous inequity aversion (i.e., $b=1$), his optimal order quantity $q^*=549=q^0$, which implies that the wholesale price contract coordinate the supply chain.

From Table 2 and Fig. 6 we can find that when the retailer’s profit is higher than the fairness reference point, with the degree of advantageous inequity aversion increases, the retailer would rather sacrifice his own interest to compensate supplier due to “sympathetic” mentality. Since the retailer’s profit decrease and the supplier’s profit increase, the gap of retailer and supplier’s profit gradually reduce. On the contrary, this will lead to promote the supply chain’s efficiency. So the decision-maker should be appropriate to make use of advantageous inequity aversion to make the channel become more harmonious.

5. Conclusions and future research

In this paper, we investigate a dyadic supply chain consisting of an inequity-averse retailer and a risk-neutral supplier with random capacity and stochastic demand. We develop supply chain model by assuming that the inequity-averse retailer has piecewise utility functions and derive the inequity-neutral retailer’s optimal order quantity based on disadvantageous inequity aversion and advantageous inequity aversion. The results
show that when the retailer is disadvantageous inequity aversion, the retailer’s optimal order quantity is decreasing in his degree of disadvantageous inequity aversion, which makes the supply chain more deviation from the optimal order quantity of the whole supply chain’s system. Thus the case can not coordinate the supply chain. However, when the retailer is disadvantageous inequity aversion, the retailer’s optimal order quantity is increasing in his degree of advantageous inequity aversion. In particular, when the retailer has extreme advantageous inequity aversion, the wholesale price contract can improve the profit of the whole supply chain and coordinate the supply chain.

Similar to any other models previously published in the literatures, the present model in our paper is also based on some assumptions. For example, our model assumes that the retailer is inequity-averse, while the supplier is risk-neutral. As a natural extension of our work, future research will consider the supplier is also inequity-averse. In addition, there may be more interesting when you consider random demand and random capacity are assumed to be dependent. Finally, more general supply chains such as multi-period models and multiple retailers and multiple suppliers’ models deserve further study.

6. ACKNOWLEDGMENTS

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7. REFERENCES


