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Optimization Design for Hub-and-Spoke Container Shipping Network Considering CO₂ Emissions

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Abstract

With the development of global economic integration, the container shipping has become the main transportation of world shipping industry. Shipping companies usually employ hub-and-spoke shipping network in order to chase for optimal economic benefits. The CO₂ emissions system of global maritime has a direct effect on shipping company's design for hub-and-spoke container shipping network. With the constraints of container flow equilibrium and hub port capacity limit, a hub-and-spoke container shipping network optimization model by considering CO₂ emissions is established. Under the measurement of container transportation costs, harbor dues and CO₂ emissions costs, the shipping company reconsider the optimal shipping routes and make decision of hub ports to be calling. The decision made by the company should aim to reduce the total shipping costs. Lagrangian relaxation algorithm is applied to solve the model. Numerical experiments show that the model and algorithm are effective. After analyzing the impact of hub ports to hub-and spoke network design, we get to the conclusion that, CO₂ emissions costs are negatively correlated with utilization ratio of hub port. And hub port capacity limit also affects the port selection decision making of shipping companies. The hub-and-spoke container shipping network optimization model by considering CO₂ has a great impact for the decision making and route optimization of the shipping companies in the future.

Keywords: Hub-and-spoke Network; CO₂ Emissions; Container Shipping Network; Lagrangian Relaxation

1. INTRODUCTION

With the development of global economic integration, the container shipping has become the main transportation of world shipping industry. In order to chase for optimal economic benefits, shipping companies usually employ hub-and-spoke shipping network (Christiansen, 2002). Maersk Line of Denmark has constructed hub-and-spoke network in Mediterranean and East Asia successively. Since then containers in Mediterranean littoral are transferred via Algeciras Port while the containers in East Asia are transferred via port of Kaohsiung. In order to implement global market expansion strategy, state-owned shipping giant COSCO and CSCL are already considering to establish own hub-and-spoke network. (Cullinane et al, 1999; Cullinane et al, 2000; Hsu et al, 2005; Jansson et al, 1987; Lim, 1998). Talley (1990) founded that economies of scale is the key to hub-and-spoke network while studying the research of container shipping. In order to reduce the shipping costs, the hub-and-spoke shipping network centralize the container flow via the hub ports to the main lines, and use transportation discount generated by the central transportation to obtain the economies of scale. Jansson et al, (1987) discussed how the port factors

influence the container shipping costs, and also pointed out the scale economy occurred in main line transportation stage, moreover diseconomies of scale happen in port calling stage. (Cullinane et al, 1999); Cullinane et al, (2000) found that reducing the layover time in the port can lower its harbor dues. Lim (1998) took into account the marginal transportation cost in main lines of hub-and-spoke network and noted that ship's unit transportation cost in main lines declines with the increase of distance. All the researches presented above proved that the transportation costs saved in main lines are sufficient to offset harbor dues (Hsu et al, 2005). Concurrently, it is also proved that cost is the real concerns of shipping company all the time (Talley, 1990). As a consequence, appropriate routines should be designed by shipping company, which centralize container ships on certain route and calls a few ports, to achieve greater economies of scale.

Shipping company should pay more attention to the influence of IMO's promotion of 'Marine Emissions Trading Scheme (METS)', as well as reconsider about finding the balance of CO₂ reduction. As the meantime of designing hub-and-spoke network, shipping company should pay more attention on impact of IMO's 'The CO₂ emissions system of global maritime' to its network. Also, it is important to reconsider about the balance of CO₂ reduction and cost controlling. Nowadays, not many researches of container shipping network design considering CO₂ emissions. Considering energy environment and cost, Winebrake et al, (2008) formulated a container multimodal transportation network model based on GIS network and discussed about container shipping network in USEC. Bauer et al. (2010) developed container multimodal transportation network model with the constraints of CO₂ emissions, and applied for container railway transportation network design. Theoretically, models above-mentioned take CO₂ emissions into account, but lack of correlation analysis of container shipping network design's cost and CO₂ emissions. With the container transportation network optimizing target of CO₂ emissions, Liao et al. (2010) developed the container shipping routing-selected model considering CO₂ emissions intensity. They also optimized container transportation network of port Taipei and Kaohsiung. Integrating CO₂ emissions with cost can form the container shipping network more scientifically. The addition of CO₂ emissions costs can makes it hard in solving the nonlinear optimization problem.

This paper is organized as follows: first, we developed a hub-and-spoke container shipping network design considering CO₂ emissions in the perspective of the whole container shipping network design. Second, aiming at reducing total costs, shipping companies need to choose the hub port, plan shipping routes as well as calculate flow rate. Third, since the model is a nonlinear 0-1 integer programming problems, Lagrangian relaxation algorithm is applied to solve the model with relaxed constraints and model decomposition. Finally, the example demonstrated the effectiveness of the simulation analysis.

2. PROBLEM STATEMENT

In a sense, container shipping can be viewed as transportation in different segments and movements between several nodes. Further, container shipping network is a network design about container flows. In our model, we assumed that the route is the path of a container pass but not a container ship. More specifically, all the hub ports and feeder ports can be seen as nodes-set, while the segments connected between nodes which are feasible in practice can be seen as collection of routes. Basic structure of hub-and-spoke container shipping network is constituted by nodes-set and collection of routes.

Usually, hub-and-spoke container shipping network is single distributing form (Wu et al, 2012; Hsu et al, 2007; Baird 2006) which means every feeder port can connect only one

hub port and needs to transit from it. Hub ports are connected by mainlines and the feeder links between feeder ports, see Figure 1. Given a finite set of ports $G=(V,E,W)$ covered a hub-and-spoke container shipping network. Among nodes-set $V=Hub \cup Spoke$, $Hub=\{J,K\}$ represents the hub ports set and $Spoke=\{I,L\}$ represents the feeder ports set. I represents the origin ports set. In routes set $E=B \cup T_1 \cup T_2$, $B=\{(j,k)|j \in J, k \in K\}$ represents the collection of mainlines while the $T_1=\{(i,j)|i \in I, j \in J\}$ and the $T_2=\{(k,l)|k \in K, l \in L\}$ represents the feeders set.

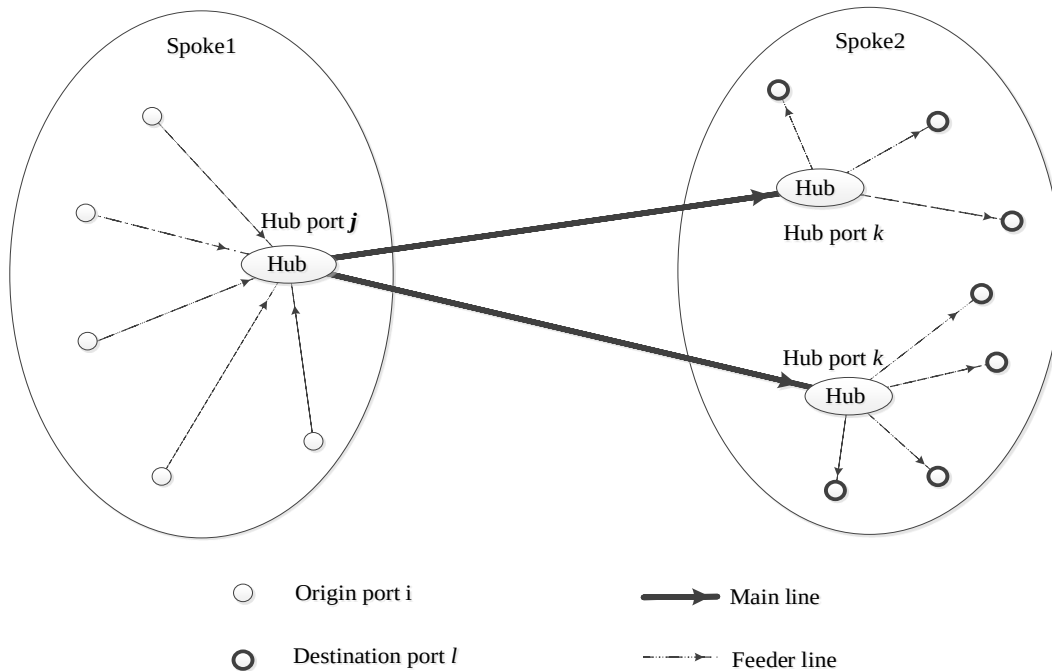


Figure 1. Hub-and-spoke container shipping network structure

$W=Q^1 \cup Q^2 \cup Q^3$ represents the container flows, in which $Q^2=\{Q^2_{jk}|j \in J, k \in K\}$ represents the container flows on mainlines, $Q^1=\{Q^1_{ij}|i \in I, j \in J\}$, $Q^3=\{Q^3_{kl}|k \in K, l \in L\}$ represents the container flows on feeder lines.

Every container is transported from origin port to destination port. The transportation need to transit through hub ports which connect feeder ports with feeder lines. If a container was transported from a hub port to another one which means $Q^1=Q^3=0$, the container flows $W=Q^2$. In reality, there exist much more hub ports in one hub-and-spoke network. But this paper aims to identify the impact of the CO2 emission. So we simplify the reality situation to the basic hub-and-spoke network.

Hub-and-spoke container shipping network requests the hub ports to connect directly while the origin ports and destination ports cannot. Transition can only happen at hub ports. Thus, every container ship is transported via hub port to the destination. (i,j,k,l) represents the nodes on shipping routes. $i \rightarrow j \rightarrow k \rightarrow l$ serve as containers' flows. Therefore, $I \times J \times K \times L$ represents the hub-and-spoke aggregate container shipping network routes (Zapfel, 2002)

The solution to hub-and-spoke container shipping network design is to find optimal shipping route $I \times J \times K \times L$ and corresponding flows Q^1, Q^2, Q^3 . The optimal scheme, actually, is aiming to reducing the shipping total costs. However, facing the container shipping trend of low-carbon of 'the CO2 emissions system of global maritime', world shipping companies are developing emission reduction measures, including carbon tax, greenhouse gas emission trading and clean production, etc. Among all kinds of emission measures, carbon tax is regarded as an economic way of convenient and efficient. Although carbon tax is a kind of environmental costs of obtaining economic benefit, it is actually a kind of CO2 emissions

costs. We need to ensure that shipping company value the CO2 emissions enough if the 'low-carbon' concept wants to integrate into container shipping network design. When developing the objective function of hub-and-spoke container shipping network design considering CO2 emissions, we need to put the CO2 emissions costs in the same position with the other costs like transportation costs and harbor dues, etc.

The container ships transporting on shipping routes belong to unit flows. Because the calling ports and the shipping routes are fixed, hub-and-spoke container shipping network follow the flow conservation. Besides, the container flows are constrained by the hub ports capacity. Every container flows of segment B, T_1, T_2 are limited by $\Lambda = \{\lambda_j^1, \lambda_k^2 | j \in J, k \in K\}$. It can generate huge harbor dues if the container flows exceed the ports capacity, according to Cullinane et al, (1999) and Cullinane et al, (2000). As a consequence, the containers may transport from one origin but take different routes to arrive the destination. Two condition should be considered which conservation of flow and hub ports capacity are.

3. MATHEMATICAL MODEL

3.1 Model Hypothesis

In order to simplify the practical issue to the mathematical model, we need to make assumptions on the hub-and-spoke network design.

Assumption 1 The hub-and-spoke container shipping network is a integrating network which cover a few main lines and feeder lines .The feeder lines link the ports in the same area while not connect the ports overseas.

Assumption 2 The hub-and-spoke container shipping network take no account of natural disaster and wars which has a stable structure.

Assumption 3 The container flows between origin and destination ports are fixed and known. It is no need to consider the limit of container ships transportation capacity.

Assumption 4 We have known all kinds of ship types of every segments and have enough ships to transport. In the paper, we do not distinguish container's ownership or whether loaded.

Assumption 5 The speed of ships of all segments are known and fixed.

Assumption 6 The hub ports use shore power which have no CO2 emissions.

3.2 Container Transport Costs

Container transport costs means costs that occurs in transportation, which can be divided into fixed cost and variable costs. Crew costs, stores, depreciation cost, insurance, and administrative costs are the main component of fixed costs. While the variable costs usually refer to energy consumption costs, including bunker age and lube costs (Shintani, 2007). We have the following binary variables : d_{ij}^1 represents the route distance of origin port i to hub port j . d_{jk}^2 represents the route distance between hub ports j and k . d_{kl}^3 represents the route distance of hub port k to destination port l . The distance is measured by nautical mile (NM). s_{jk} is the fixed cost per unit of main lines (USD/NM). r_{ij}, t_{jk} represent the fixed cost per unit of feeder lines (USD/NM), respectively. b_{jk} is the energy consumption per unit of main lines (kg/TEU·NM). a_{ij}, c_{kl} stand for the energy consumption per unit of feeder lines (kg/ TEU·NM), respectively. ρ is introduced to represents the energy prices (USD/kg). The considered objective transportation cost function $C_{transportation}$ for the hub-and-spoke container shipping network:

$$C_{transportation} = \sum_{i \in I, j \in J, k \in K, l \in L} \left\{ (r_{ij} + \rho a_{ij} Q_{ij}^1) d_{ij}^1, (s_{jk} + \rho b_{jk} Q_{jk}^2) d_{jk}^2, (t_{kl} + \rho c_{kl} Q_{kl}^3) d_{kl}^3 \right\} \quad (1)$$

In the formula above, Q_{ij}^1 represents the container flowing designed. And the $(r_{ij} + \rho a_{ij} Q_{ij}^1) d_{ij}^1$ is the transportation costs of Q_{ij}^1 on feeder lines (i, j) . Similarly, $(s_{jk} + \rho b_{jk} Q_{jk}^2) d_{jk}^1$ stands for the transportation costs of Q_{jk}^2 on feeder lines (j, k) ; $(t_{kl} + \rho c_{kl} Q_{kl}^3) d_{kl}^3$ stands for the transportation costs of Q_{kl}^3 on feeder lines...

3.3 Harbor Dues

Harbor dues is the transfer costs of the container ships happens in the hub ports. It can be classified as clearance fee and handling charges. Clearance fee including Pilotage dues, towage fee and berthage, etc. The handling charges, facility charges and CY charges belong to handling charges.

Harbor dues is an important cost factor of economies of scale in the Hub-and-spoke container shipping network. In practice, the container ships need to queue in at anchorage waiting for entering the harbor when caught in a port congestion. The port overstock can produce great amount expenses. In the previous study (Lim, 1998; McLellan, 1997), it is easy to find the relationship between port capacity and harbor dues, while it's hard to use port capacity function to estimate harbor dues. Cullinane et al, (1999) and Cullinane et al, (2000) pointed that harbor dues has a positive correlation with lay-time. Nevertheless, the lay-time is inversely proportional to port capacity. Then we can get the relationships of them: harbor dues $\propto$ lay-time $\propto$ 1/port capacity.

μ_j, μ_k (USD) represent the inward charges of hub ports j, k , respectively. ν_j, ν_k (USD/TEU) represent the terminal charges per unit of hub ports j, k . λ_j^1, λ_k^2 represent the port capacity of hub ports j, k , respectively. Suppose $C_{hub} \propto \lambda^{-2/3}$, then harbor dues function of hub-and-spoke container shipping network C_{hub} can be expressed as:

$$C_{hub} = \bigcup_{j, k}^{j \in J, k \in K} \left\{ \left(\mu_j + \nu_j \sum_{k=1}^p Q_{jk}^2 \right) (1 + \lambda_j^{1-2/3}), \left(\mu_k + \nu_k \sum_{l=1}^q Q_{kl}^3 \right) (1 + \lambda_k^{2-2/3}) \right\} \quad (2)$$

In the formula, $\left(\mu_j + \nu_j \sum_{k=1}^p Q_{jk}^2 \right) (1 + \lambda_j^{1-2/3})$ represent the container ships of Q_{jk}^2 port capacity at hub port j . Similarly, $\left(\mu_k + \nu_k \sum_{l=1}^q Q_{kl}^3 \right) (1 + \lambda_k^{2-2/3})$ stand for the container ships of Q_{kl}^3 port capacity at hub port k .

3.4 CO₂ Emission Cost

Measurement and selection of CO₂ emissions and carbon tax rate are involved in the calculation of CO₂ emission cost. CO₂ emission cost = CO₂ emissions $\times$ carbon tax rate. We need to know energy consumption and CO₂ emission factor in the transportation if we calculate the CO₂ emissions. It's probably worth pointing out that CO₂ emission factor refers to the amount of energy consumption unit mass produced by the greenhouse gases transferring into CO₂. CO₂ emission factor is one important parameter of describing energy greenhouse gas emission characteristics (Kim et al, 2012). A carbon tax is a tax levied on the CO₂ emission of fuels (e.g. natural gas, refined oil).

In hub-and-spoke container shipping network, k represents the carbon tax rate (USD/kg), CO₂ emission factor is represented by θ . CO₂ emission cost $C_{emission}$, are represented in the formula as

$$C_{emission} = \bigcup_{i, j, k, l}^{i \in I, j \in J, k \in K, l \in L} \left(\kappa \theta a_{ij} Q_{ij}^1 d_{ij}^1, \kappa \theta b_{jk} Q_{jk}^2 d_{jk}^2, \kappa \theta c_{kl} Q_{kl}^3 d_{kl}^3 \right) \quad (3)$$

In the equation, $\kappa \theta a_{ij} Q_{ij}^1 d_{ij}^1$ represents CO₂ emission cost of designed container flows Q_{ij}^1 on

feeder lines (i, j) . $\kappa\theta b_{jk}Q_{jk}^2d_{jk}^2$ represents CO2 emission cost of designed container flows Q_{jk}^2 on feeder lines (j, k) . $\kappa\theta c_{kl}Q_{kl}^3d_{kl}^3$ represents CO2 emission cost of designed container flows Q_{kl}^3 on feeder lines (k, l) in the same way.

3.5 Model Development

The hub-and-spoke container shipping network design considering CO2 emissions can be seen in brief as follows. Given a finite hub-and-spoke container shipping network $G=(V, E, W)$. $V=Hub \cup Spoke=\{I, J, K, L\}$ is the set of nodes. $E=B \cup T_1 \cup T_2$ represents the set of segments and $W=Q^1 \cup Q^2 \cup Q^3$ is the container flows. The capacity of hub ports set can be express as $\Lambda=\{\lambda_j^1, \lambda_k^2\}$. $I=\{i|i=1, 2, \dots, m\}$, $J=\{j|j=1, 2, \dots, n\}$, and $K=\{k|k=1, 2, \dots, p\}$, $L=\{l|l=1, 2, \dots, q\}$. Our targets are going to verify two things: the routes set $I \times J \times K \times L$ of hub-and-spoke container shipping network and the container flows Q^1, Q^2, Q^3 on the routes. We introduce 0-1 binomial distribution x_{ij} , y_{jk} , z_{kl} to describe whether node j, k is chosen as hub ports of calling, and whether node l is chosen as origin port.

$$x_{ij} = \begin{cases} 1, & \text{feeder transportation } i \rightarrow j \text{ call at node } j \\ 0, & \text{else} \end{cases}, \quad (4)$$

$$y_{jk} = \begin{cases} 1, & \text{mainline transportation } j \rightarrow k \text{ call at node } k \\ 0, & \text{else} \end{cases}, \quad (5)$$

$$z_{kl} = \begin{cases} 1, & \text{feeder transportation } k \rightarrow l \text{ call at node } l \\ 0, & \text{else} \end{cases}, \quad (6)$$

We proposed hub-and-spoke container shipping network optimizing model considering CO2 emission with minimum shipping cost (including container transport cost $C_{transportation}$, harbor dues C_{Hub} , and CO2 emission cost $C_{emission}$) are showed as follows,

$$\begin{aligned} \text{Min } C = & \sum_{i=1}^m \sum_{j=1}^n \left(r_{ij} + \rho a_{ij} Q_{ij}^1 + \kappa \theta a_{ij} Q_{ij}^1 \right) d_{ij}^1 x_{ij} + \\ & \sum_{j=1}^n \sum_{k=1}^p \left(\left(s_{jk} + \rho b_{jk} Q_{jk}^2 + \kappa \theta b_{jk} Q_{jk}^2 \right) d_{jk}^2 y_{jk} + \left(\mu_j + \nu_j \sum_{k=1}^p Q_{jk}^2 \right) \left(1 + \lambda_j^{1-2/3} \right) y_{jk} \right) + \\ & \sum_{k=1}^p \sum_{l=1}^q \left(\left(t_{kl} + \rho c_{kl} Q_{kl}^3 + \kappa \theta c_{kl} Q_{kl}^3 \right) d_{kl}^3 z_{kl} + \left(\mu_k + \nu_k \sum_{l=1}^q Q_{kl}^3 \right) \left(1 + \lambda_k^{2-2/3} \right) z_{kl} \right), \end{aligned} \quad (7)$$

s.t.

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i \in I, \quad (8)$$

$$\sum_{k=1}^p z_{kl} = 1 \quad \forall l \in L, \quad (9)$$

$$\sum_{j=1}^n y_{jk} = 1 \quad \forall k \in K, \quad (10)$$

$$\sum_{i=1}^m Q_{ij}^1 x_{ij} = \sum_{k=1}^p Q_{jk}^2 y_{jk} \quad \forall j \in J, \quad (11)$$

$$\sum_{l=1}^q Q_{kl}^3 z_{kl} = \sum_{j=1}^n Q_{jk}^2 y_{jk} \quad \forall k \in K, \quad (12)$$

$$\sum_{i=1}^m Q_{ij}^1 x_{ij} \leq \lambda_j^1 \quad \forall j \in J, \quad (13)$$

$$\sum_{l=1}^q Q_{kl}^3 z_{kl} \leq \lambda_k^2 \quad \forall k \in K, \quad (14)$$

$$\sum_{k=1}^p Q_{jk}^2 y_{jk} \leq \lambda_j^1 \quad \forall j \in J, \quad (15)$$

$$\sum_{j=1}^n Q_{jk}^2 y_{jk} \leq \lambda_k^2 \quad \forall k \in K, \quad (16)$$

$$Q_{ij}^1, Q_{jk}^2, Q_{kl}^3 > 0 \quad \forall i \in I, \forall j \in J, \forall k \in K, \quad \forall l \in L, \quad (17)$$

$$x_{ij}, y_{jk}, z_{kl} \in \{0, 1\} \quad \forall i \in I, \forall j \in J, \forall k \in K, \quad \forall l \in L, \quad (18)$$

The constraints (8) and (9) are the port selection constraints of feeder lines transportation $(i,j),(k,l)$. Constraint (10) is the port selection constraint of mainline transportation (j,k) . The constraints (11) and (12) are container flow conservation, which means for hub ports j,k , the container flows entering the port equal to the flows leaving the port. Constraints (13)-(16) are the hub port capacity constraints. Constraint (13) and (14) show that the container flow transported on feeder line $(i,j),(k,l)$ cannot exceed hub port j,k 's capacity. Similarly, Constraint (15) and (16) mean that container flow transported on mainline (j,k) cannot exceed hub port j,k 's capacity. And formula (17) is the non-negative constraint of demand.

3.6 Solution Method

The main challenge in solution is that the formulas (7)-(18) are nonlinear mixed 0-1 integer programming problem. So, it is a NP-hard problem and cannot solve in polynomial time (Feng, 2012). Problems may appear if we use heuristic method like simulated annealing method or tabu search method. The problems are usually long-time searching and low precision optimum solution. Compared with methods above mentioned, Lagrangian relaxation algorithm can get optimum solution or even know the how much difference the solution with optimum one through controlling relaxation process. The problem can be separated into several subproblems when complex constraints was absorbed into objective function as penalty term by means of lagrangian multiplier (Fisher, 1981)0, which has advantage of calculation efficiency. Lagrangian relaxation algorithm has superiority when solving problems with plenty of variables.

In hub-and-spoke container shipping network, any route of a container can be expressed as (i,j,k,l) . And the routes design are split into dependent segment (j,k) , (i,j) , (k,l) . As a consequence, the formulas (7)-(18) can be seen as combinatorial optimization of mainline B and feeder lines T_1, T_2 . Container flows conservation constraints (11) and (12) are

complex constraints of solution, which are the key constraints of balancing mainline and feeder line transportation. Through introducing Lagrangian multiplier $\delta_j, j=1,2,\dots,n$ and $\phi_k, k=1,2,\dots,p$, Lagrangian relaxation is applied on constraints (11) and (12) and the model will be established after plugging the two constraints into objective (7)

$$\left\{ \begin{array}{l} \text{Min } C_{LR}(\delta_j, \phi_k) = \sum_{i=1}^m \sum_{j=1}^n \left((r_{ij} + \rho a_{ij} Q_{ij}^1 + \kappa \theta a_{ij} Q_{ij}^1) d_{ij}^1 x_{ij} + \delta_j Q_{ij}^1 x_{ij} \right) + \\ \sum_{j=1}^n \sum_{k=1}^p \left((s_{jk} + \rho b_{jk} Q_{jk}^2 + \kappa \theta b_{jk} Q_{jk}^2) d_{jk}^2 y_{jk} - (\delta_j + \phi_k) Q_{jk}^2 y_{jk} + (\mu_j + \nu_j Q_{jk}^2) (1 + \Lambda_j^{1-2/3}) y_{jk} \right) + \\ \sum_{k=1}^p \sum_{l=1}^q \left((t_{kl} + \rho c_{kl} Q_{kl}^3 + \kappa \theta c_{kl} Q_{kl}^3) d_{kl}^3 z_{kl} + \phi_k Q_{kl}^3 z_{kl} + (\mu_k + \nu_k Q_{kl}^3) (1 + \lambda_k^{2-2/3}) z_{kl} \right), \\ \text{s.t. formulas(8)~(10), formulas(13)~(18)} \end{array} \right. \quad (19)$$

The Lagrangian relaxation formula (19) of formulas (7) - (18) can be divided into three dependent sub-problems as follows,

$$\left\{ \begin{array}{l} \text{Min } C_{LR1}(\delta_j, \phi_k) = \sum_{k=1}^p \sum_{l=1}^q \left((t_{kl} + \rho c_{kl} Q_{kl}^3 + \kappa \theta c_{kl} Q_{kl}^3) d_{kl}^3 z_{kl} + \phi_k Q_{kl}^3 z_{kl} \right. \\ \left. + (\mu_k + \nu_k Q_{kl}^3) (1 + \lambda_k^{2-2/3}) z_{kl} \right) \\ \text{s.t. formulas(9),(14),(17),(18).} \end{array} \right. \quad (20)$$

$$\left\{ \begin{array}{l} \text{Min } C_{LR2}(\delta_j, \phi_k) = \sum_{j=1}^n \sum_{k=1}^p \left((s_{jk} + \rho b_{jk} Q_{jk}^2 + \kappa \theta b_{jk} Q_{jk}^2) d_{jk}^2 y_{jk} - (\delta_j + \phi_k) Q_{jk}^2 y_{jk} + \right. \\ \left. (\mu_j + \nu_j Q_{jk}^2) (1 + \Lambda_j^{1-2/3}) y_{jk} \right) \\ \text{s.t. formulas(10),(15)-(18).} \end{array} \right. \quad (21)$$

$$\left\{ \begin{array}{l} \text{Min } C_{LR3}(\delta_j, \phi_k) = \sum_{i=1}^m \sum_{j=1}^n \left((r_{ij} + \rho a_{ij} Q_{ij}^1 + \kappa \theta a_{ij} Q_{ij}^1) d_{ij}^1 x_{ij} + \delta_j Q_{ij}^1 x_{ij} \right) \\ \text{s.t. formulas(8),(13),(17),(18).} \end{array} \right. \quad (22)$$

Among all the formulas above, sub problem (20) relates to selected variable z_{kl} from hub k to destination l . And the sub problem (22) relates to selected variable x_{ij} from hub i to destination j . Those two all belong to feeder lines transportation design. While sub problem (21), which belongs to main line transportation design, relate to selected variable y_{jk} . Compared with formulas (7)-(18), the scale of solving sub problems (20)-(22) is apparently smaller and has low complexity. A few iterations are needed when the problems approach the optimum solution. Moreover, we can acquire the chosen hub k of the feeder line transportation using z_{kl} in (20), which is the condition of (21). It is clear that (20)-(22) are consecutive.

Apparently, $C_{LR}(d_j, \phi_k) < C$. According to Lagrangian multiplier and constraints, we come to dual problem of (7) - (18). That is

$$C_{LD}(\delta_j, \phi_k) = \text{Max} \left\{ C_{LR1}(\delta_j, \phi_k) + C_{LR2}(\delta_j, \phi_k) + C_{LR3}(\delta_j, \phi_k) \right\} \quad (23)$$

Taking no consideration about the limit of constraints (11) - (12), dual solutions of (23) may not be the feasible result of (7)-(18). Under the limit of hub port capacity, this step ensure each container will transport on main lines and feeder lines from origin to destination at least. At the same time, dual solutions provide (7)-(18) with the lower bound

$C_{LR}(d_j, \phi_k)$ of objective function. Using consecutiveness of (20)-(22) and $\sum_{l=1}^q Q_{kl}^3 z_{kl}$ as well as $\sum_{k=1}^p Q_{jk}^2 y_{jk}$, we work out feasible result of (7)-(18). Finally, we can get upper bound $C_{LU}(d_j, \phi_k)$ after plugging feasible result into objective function (7). By adopting sub-gradient algorithm, Lagrangian multipliers $\delta_j, j=1,2,\dots,n$ and $\phi_k, k=1,2,\dots,p$ are updated. Upper and lower bound of objective function are getting precise, and optimum solution of (7)-(18) is acquired. Given initial multipliers d_j^0 and ϕ_k^0 , rules of updating are as follows,

$$\delta_j^{\tau+1} = \delta_j^{\tau} + \eta^{\tau} \left(\sum_{k=1}^p Q_{jk}^2 y_{jk} - \sum_{i=1}^m Q_{ij}^1 x_{ij} \right) \quad (24)$$

$$\phi_k^{\tau+1} = \phi_k^{\tau} + \eta^{\tau} \left(\sum_{j=1}^n Q_{jk}^2 y_{jk} - \sum_{l=1}^q Q_{kl}^3 z_{kl} \right), \quad (25)$$

In the formula, τ represents iteration, iteration steps can be expressed as

$$\eta^{\tau} = \frac{\omega^{\tau} [C_{LU}(\delta^{\tau}, \phi^{\tau}) - C_{LD}(\delta^{\tau}, \phi^{\tau})]}{\sum_{j=1}^m s(\delta_j^{\tau})^2 + \sum_{k=1}^p s(\phi_k^{\tau})^2} \quad (26)$$

ω^{τ} represents step parameter, and set $\omega^0=2$ as well as updating rule $\omega^{\tau+1}=0.98\omega^{\tau}$; $s(d_j^{\tau})$ and $s(\phi_k^{\tau})$ represent multipliers δ_j and ϕ_j corresponding sub-gradient, that is

$$s(\delta_j^{\tau}) = \sum_{i=1}^m Q_{ij}^1 x_{ij}^{\tau} - \sum_{k=1}^p Q_{jk}^2 y_{jk}^{\tau} \quad (27)$$

$$s(\phi_k^{\tau}) = \sum_{l=1}^q Q_{kl}^3 z_{kl}^{\tau} - \sum_{j=1}^n Q_{jk}^2 y_{jk}^{\tau} \quad (28)$$

Relevant theory of the sub-gradient optimization algorithm can be found in Xing (2005)'s paper.

Algorithm steps based on Lagrangian relaxation are as follows:

Step 1 Choose a pair of initial Lagrange multiplier $(\delta_j^{\tau}, \phi_k^{\tau}), \tau=0$;

Step 2 We get sub-gradient $s(\delta_j^{\tau})$ and $s(\phi_k^{\tau})$ according to $(\delta_j^{\tau}, \phi_k^{\tau})$;

Step 3 If $(\delta_j^{\tau}, \phi_k^{\tau})$ meets any requirements below, it comes to optimum solution. The operation stops. Otherwise, turn to Step 4;

$C_{LU}(\delta_j, \phi_k) - C_{LD}(\delta_j, \phi_k) \leq$ the allowable deviation of value by default;

For $\forall j, k, s(\delta_j^{\tau})=0, s(\phi_k^{\tau})=0$;

$\eta^{\tau} \leq$ the default step length limit;

Iterations τ =the default maximum number of iterations.

Step 4 Updated $(\delta_j^{\tau+1}, \phi_k^{\tau+1})$ according to the above rules, and turn to Step 2.

4. SIMULATION

In order to demonstrate the hub-and-spoke container shipping network design considering CO₂ emissions, we need to conduct simulation.

Based on the hub-and-spoke network of two area, the port location generated on the coordinates randomly. Origin port $i=1,2,\dots,10$, and destination port $l=1,2,\dots,10$. $j=1,2,\dots,4$ and $k=1,2,\dots,4$ represent hub ports of each district. Route distance between ports can be obtained according to the port coordinates. See Figure 2-3.

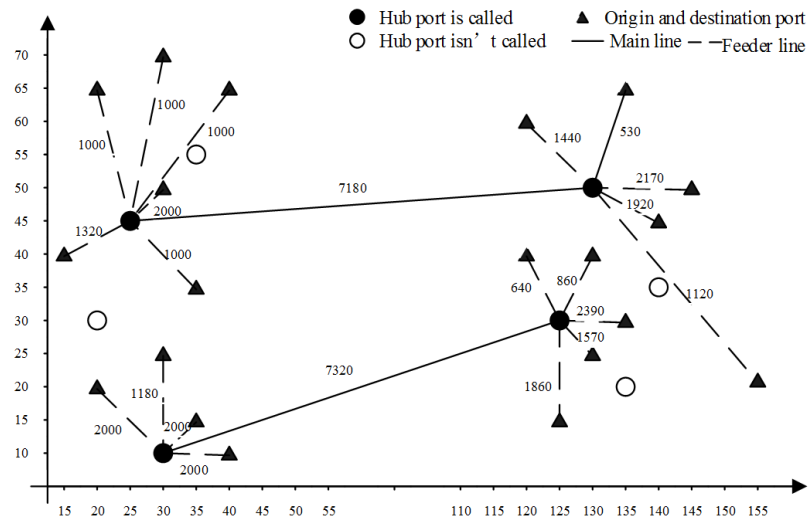


Figure 2. Hub-and-spoke container shipping network design considering CO₂ emissions cost not considered

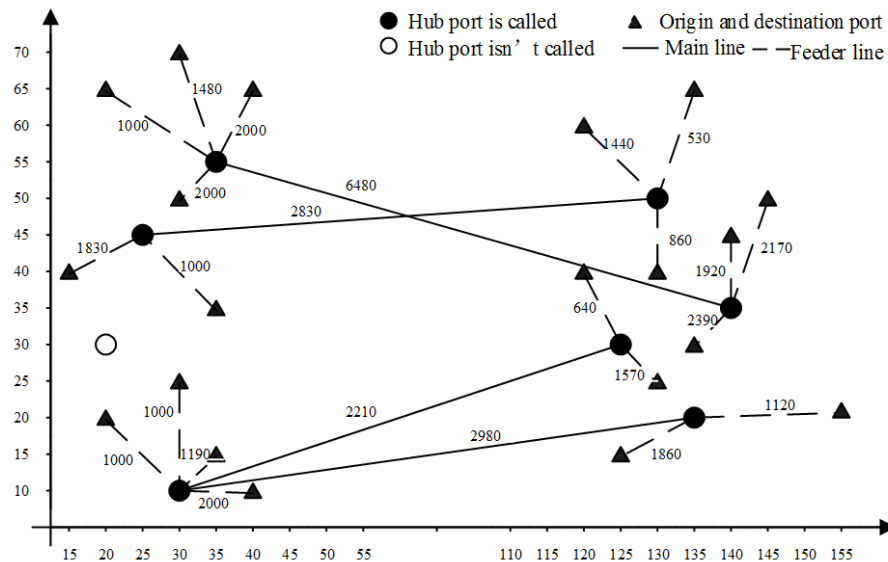


Figure 3. Hub-and-spoke container shipping network design considering CO₂ emissions cost considered

Here, we made some assumptions. Suppose that the forecasting container shipping demand by a shipping company is:

$$Q_{kl}^3 = [1860, 640, 1440, 860, 1570, 530, 1920, 2390, 1120, 2170]$$

The hub port capacity $\lambda_j^1 = [7000, 6000, 7600, 6500]$, $\lambda_k^2 = [7500, 7400, 6500, 7000]$. There is competition between hub ports, and there are not so many differences of charging inward charges and terminal charges. μ_j, μ_k satisfies $U[110, 130]$ and v_j, v_k satisfies $U[6, 8]$. Usually, small container ships transport on feeder lines and fixed cost per unit r_{ij}, t_{kl} satisfies $U[290, 320]$. While large container ships transportation on main line and fixed cost per unit s_{jk} satisfies $U[490, 510]$. Normally, energy consumption per unit of large container ships is higher than small container ships (Tao, 2006). We can find the relationships, $b_{jk} = \psi a_{ij}$ and $b_{jk} = \psi c_{kl}$. Take $\psi = 1.3$, and energy price $\rho = 1$.

4.1 The Impact of CO2 Emission to Hub-and-spoke Network Design

$k\theta = 0.25$ when take consideration of CO2 emission costs. $k\theta = 0$ is the condition of no considering about CO2 emission costs. Hub-and-spoke container shipping network optimization models are established respectively. Calculated by MATLAB R2013a programming, the result shows in table 1. Hub port utilization can be express as ratio of the sum of container flow and the sum of calling hub port capacity, which reflect the agglomeration effect of hub ports in the network. Table 1 indicates that, utilization of port j, k are 0.9932 and 0.9732, respectively, when $k\theta = 0$. When $k\theta = 0.25$, utilization of port j, k are just 0.6872 and 0.5106, which reflect that CO2 emission costs has negative correlation with hub port utilization. Table 1 shows that, Lagrangian relaxation algorithm has high calculating precision, and the deviation of upper and lower bound of objective function are both under 2%.

Table 1 The results based on Lagrangian relaxation algorithm

	utilization of port j	utilization of port k	upper bound	lower bound	the deviation of upper and lower bound	iterations
CO ₂ emissions cost considered	0.6872	0.5106	8011703	7853137	1.98%	38
CO ₂ emissions cost not considered	0.9932	0.9732	6695612	6580875	1.72%	31

1) $k\theta = 0$. A large part of container ships call the hub ports of high efficiency and transit capacity, which reflect agglomeration effect of hub ports in the network. Two hub ports in each area are chosen as transit station. See in Figure 2.

Because of the high cost of harbor dues, shipping companies will choose hub ports of largest capacity (high efficiency and transit capacity) as far as possible, with no considering about CO2 emission cost. However, limited by the capacity, some container ships have to call hub ports of the secondary large capacity, in order to achieve scale economy of hub-and-spoke shipping network.

2) $k\theta = 0.25$. Agglomeration effect of hub ports in network is weakened due to the dispersity of calling ports. Origin choose 3 hub ports to transfer while destination choose 4 ports to transfer in two ocean area. See the Figure 3.

Thus it can be seen that, CO2 emission cost has cause conflict in designing hub-and-spoke shipping network design. When considering about CO2 emission cost, the high cost of container transportation and CO2 emissions cost shift shipping company's attention to shorten the routes distance. In order to realize the optimal design of hub-and-spoke container shipping network design, more hub ports are selected to reduce shipping route distance.

4.2 The Impact of Hub Ports to Hub-and spoke Network Design

From harbor dues cost function, the harbor dues cost $\propto$ layover time $\propto 1/\text{port capacity}$. The hub ports could enlarge the port capacity through investing relevant resources, which

reduce berth time and impel the ships calling at ports. So, hub ports capacity play an important role in the decision of choosing ports.

Considering about the CO2 emission cost, take $k\theta=0.25$ and keep the other parameters constant. Expand the hub ports capacity A_j^1, A_k^2 at the same time. Take the multiplier $\xi=1, 1.4, 2.4$, and design for different scale of i, j, k, l in hub-and spoke shipping network. The result shows in table 2.

With the increasing of hub ports capacity multiplier ξ , the average amount of hub ports chosen by container ships has decrease from (3.6, 3.8) to (1.8, 2.2). There-into, the utilization scope of hub port j increase from (0.6557, 0.8788) to (0.5556, 0.9028), while hub port k increase from (0.6655, 0.8841) to (0.4554, 0.9848). Meantime, the average utilization of hub ports j, k decrease a bit due to the expanding of capacity, which from (0.7764, 0.7897) to (0.7503, 0.6537). The data indicates that the expanding of capacity is benefit to the structural stability, and the hub-and spoke network can express its economies of scale immensely. In addition, the decrease of the ratio of harbor dues to shipping costs and increase of ratio of transportation cost and CO2 emission cost to shipping costs, show that the expanding of capacity can decrease harbor dues, and also make up the disadvantage of route distance. It can attract more container ships for the ports.

Table 2 Hub-and-spoke container shipping network design based on different port capacity

Expanding multiplier i, j, k, l	Utilization of port j	Utilization of port k	Selected number of port j	Selected number of port k	The ratio of harbor dues to shipping costs	The ratio of transportation cost to shipping costs	The ratio of CO2 emission cost to shipping costs	Iterations
$\xi=1$								
6,3,3,6	0.8025	0.8497	3	3	0.1899	0.6623	0.1478	32
8,3,3,8	0.8788	0.8841	3	3	0.1904	0.6610	0.1487	41
10,4,4,10	0.6827	0.6655	3	4	0.1837	0.6670	0.1493	46
12,4,4,12	0.7557	0.8230	4	4	0.1923	0.6592	0.1485	48
15,5,5,15	0.7625	0.7263	5	5	0.1946	0.6570	0.1483	75
Min	0.6557	0.6655	3	3	0.1837	0.6610	0.1478	32
Mean	0.7764	0.7897	3.6	3.8	0.1902	0.6613	0.1485	48.4
Max	0.8788	0.8841	5	5	0.1946	0.6670	0.1493	75
$\xi=1.4$								
6,3,3,6	0.8217	0.8844	2	2	0.1562	0.6863	0.1575	26
8,3,3,8	0.9006	0.9502	2	2	0.1648	0.6790	0.1562	39
10,4,4,10	0.7508	0.7221	3	3	0.1517	0.6905	0.1578	33
12,4,4,12	0.6349	0.6094	3	4	0.1612	0.6824	0.1564	51
15,5,5,15	0.7549	0.7061	4	4	0.1568	0.6858	0.1574	65
Min	0.6349	0.6094	2	2	0.1517	0.6790	0.1562	26
Mean	0.7726	0.7744	2.8	3.0	0.1581	0.6848	0.1571	40.0
Max	0.9006	0.9502	4	4	0.1648	0.6905	0.1578	65
$\xi=2.4$								
6,3,3,6	0.9028	0.9848	1	1	0.1139	0.7169	0.1692	28
8,3,3,8	0.8745	0.5543	1	2	0.1121	0.7221	0.1658	19
10,4,4,10	0.8030	0.6822	2	2	0.1116	0.7205	0.1678	40
12,4,4,12	0.5556	0.4554	2	3	0.1232	0.7058	0.1710	60
15,5,5,15	0.6155	0.5920	3	3	0.1480	0.6915	0.1606	55
Min	0.5556	0.4554	1	1	0.1116	0.6915	0.1606	19
Mean	0.7503	0.6537	1.8	2.2	0.1218	0.7114	0.1669	36.8
Max	0.9028	0.9848	3	3	0.1480	0.7221	0.1678	60

5. CONCLUSION

In this paper, we analyze the hub-and-spoke container shipping network design from the perspective of shipping company. The influence of economies of scale, calling hub ports and CO₂ emission to shipping network design, combining with the measurement of containers flow conservation constraints and capacity limits, we proposed hub-and-spoke container shipping network design considering CO₂ emissions. Based on separable structures of the model, it was solved by Lagrangian relaxation algorithm. The simulation showed that: (1) CO₂ emission cost has negative correlation with hub ports utilization. Without regard of CO₂ emission cost, container ships gather at hub ports of largest capacity. While taking account of CO₂ emission cost, container ships are calling at several hub ports separately in order to connect the feeder ports. (2) The hub ports capacity influence the choice of shipping company. Expanding capacity and decrease in the harbor dues can cover the shortage of route distance. Nowadays, just few researches are concerning container shipping network considering CO₂ emission. More work is needed when concerned about the reality situation of which are more complex, dynamic and uncertain.

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